Using Hidden Markov Model to Detect Macro-economic Risk Level

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ABSTRACT

In this paper, inspired by Moody's BET model, a stochastic hidden Markov model is constructed to detect the macro-economic risk states hidden in the corporate default data. The observed default statistics are from four geographic regions, namely Asia-Pacific, Europe, the U.S. and the globe as a whole. The EM algorithm is applied to estimate parameters in each model, where the associated standard errors are computed using the Monte Carlo method. The validity of the binomial distribution assumption is checked by conducting the Chi-square goodness-of-fit test. When compared with the historical recession and expansion periods, most of the estimated risk-switching processes are in accord with the actual fluctuations in the macro-economy.

Keywords: hidden Markov model; credit default analysis; EM algorithm; Monte Carlo method.

1. INTRODUCTION

The recent global financial crisis has forced academics to take a renewed interest in discovering or forecasting risk levels in the macro-economy by modeling with financial market data. During the past 10 years, Moody's and Standard & Poor's have been using corporate default history in different geographical regions to assess the performance of their long-term corporate rating mechanisms annually. In 1996, Moody's proposed a Binomial Expansion Technique (BET) to measure the portfolio credit risk within a fixed time period (Moody's, 1996). For simplicity, they assumed the bonds in a portfolio defaulted independently and discovered that default counts followed a binomial distribution over a fixed time interval. In their research, a 'Diversity Score' was calculated to account for the interaction effects of the member bonds in a portfolio, indicating that the risk probability of a portfolio is inversely proportional to its degree of diversity. Being a discrete model, Moody's BET is useful in modeling the occurrence of default events in a given period of time but not at each time point. Hence, in Davis and Lo's paper (2001), they introduced the dynamic 'Enhanced Risk Model' to measure the effects of infectious defaults with a stochastic process. Following their theory, the occurrence of a default event in a portfolio would 'enhance' the default probability of the remaining surviving bonds by a factor K. Before settling back to 'normal risk', the portfolio would stay in the 'enhanced risk' state where the default time is exponentially distributed. In terms of empirical study, Gilchrist et al. (2009) measured the influences of corporate credit spread shock on business cycles based on Bayesian ML estimation. In this paper, a similar stochastic approach, the hidden Markov model (HMM), was applied to explain the fluctuations in the macro-economy. Inspired by Davis & Lo (2001) and Rabiner (1989), the two risk states in our model are recognized as hidden macro-economic risk levels and follow a two-state Markov-switching process over a specified time period. In addition, following Giampieri et al. (2005), a Monte Carlo method is used to estimate the standard errors associated with the parameters generated by the Expectation-Maximization (EM) algorithm.

The remainder of this paper is organized as follows: Section 2 will cover the methodology used and in Section 3, simulation results will be presented to verify the reliability of the whole estimation process. Data used in this paper will be described in Section 4. In Section 5, parameter estimation results, along with the associated standard errors are shown in tables and detailed analysis with real business cycles will follow. A Chi-square goodness-of-fit test will be conducted to test the model assumption in Section 6. The remaining Sections 7 and 8 will suggest directions for future studies and make a final conclusion, respectively.

2. METHODOLOGY

2.1 Model Description

The model used in this paper is a two-state HMM, where the states are denoted as 1 and 2, representing 'normal risk state' and 'enhanced risk state', respectively. In this case, the initial probability of being at either state is stored in matrix π. Following the BET by Moody's (1996), in state 1 the default events in each time step follow a binomial distribution with parameter p and the probability of observing D default events is P_1. To incorporate the effects of one default event on the remaining surviving bonds, the parameter in P_2 is calculated by multiplying p with a factor $K \geq 1$ Specifically, N represents the number of bonds remaining 'alive' in a portfolio.

$$P_1(D) = \binom{N}{D} p^D (1-p)^{N-D} , \qquad (1)$$

$$P_2(D) = \binom{N}{D}(Kp)^D(1-Kp)^{N-D} \ , \qquad (2)$$

The above two equations compute the probability of observing all possible counts of default events and thus constitute the emission matrix B. In terms of the construction of transition matrix A, the following parameters are used:

$$A = \begin{bmatrix} a_{11} & 1-a_{11} \\ 1-a_{22} & a_{22} \end{bmatrix}, \qquad (3)$$

where a_{11} denotes the probability of staying in state 1 and a_{22} denotes the probability of staying in state 2. Once given the actual observation sequence and HMM, the optimal hidden state sequence can be computed by the Viterbi algorithm. According to Rabiner (1989), $\delta_t(i)$ is defined as the highest probability of observing the previous t observations at state i, with one hidden state sequence. In this paper, D denotes the actual observation; Q denotes the hidden state sequence while $\lambda\,(A, B, \pi)$ summarizes all the parameters used in HMM. The essence of the Viterbi algorithm is to maximize P $(Q|D, \lambda)$ and use backtracking steps to retrieve the state path. In this HMM, all observations are treated as statistically independent.

2.2 Parameter Estimation

One essential problem of the HMM is: Given the actual observation sequence, how can P $(D|\lambda)$ be maximized by estimating the appropriate parameters in the model? Referring to Rabiner (1989), the EM algorithm can generate the estimators in the HMM which locally maximize P $(D|\lambda)$. In this algorithm, the forward variable $\alpha_t(i)$ is defined as the probability of observing the sequence before reaching state i at time t. Similarly, the backward variable $\beta_t(i)$ denotes the probability of observing the sequence till the end, after being at state i at time t. Hence, the following variables can be constructed in terms of the above two:

$$\tau_t(i,j) = \frac{\alpha_t(i)\cdot A_{ij}\cdot B_{jD_{t+1}}\cdot \beta_{t+1}(j)}{\sum_{\substack{1\le i\le N\\1\le j\le N}} \alpha_t(i)\cdot A_{ij}\cdot B_{jD_{t+1}}\cdot \beta_{t+1}(j)} \ , \qquad (4)$$

$$\gamma_t(i) = \frac{\alpha_t(i)\cdot \beta_t(i)}{\sum_{i=1}^{N} \alpha_t(i)\cdot \beta_t(i)} = \sum_{j=1}^{N} \tau_t(i,j) \ , \qquad (5)$$

where $\tau_t(i,j)$ represents the probability of being in state i at time t and in state j at the next time point, given the model and observation sequence; $\gamma_t(i)$ represents the probability of being in state i at time t. All these defined variables are then implemented into the EM re-estimation procedure. The difference between two consecutive likelihood functions eventually converges to a critical value, namely 10^{-15} in our case. In the binomial distribution, the maximum likelihood estimator of p is calculated as p = $\frac{\text{Average number of 'successful' trials}}{\text{Total number of trials}}$. Another important step in realizing this algorithm is 'scaling' since the forward and backward variables can be extremely small when t gets larger. For simplicity, $\alpha_t(i)$ and $\beta_t(i)$ are multiplied by $C_{1t} = \frac{1}{\sum_{i=1}^{N} \alpha_t(i)}$ and $C_{2t} = \frac{1}{\sum_{i=1}^{N} \beta_t(i)}$ respectively to prevent the calculation from exceeding the precision range of the computer (Detailed procedures please see Rabiner's paper (1989)).

2.3 Standard Error Computation

Due to the inability of EM algorithm to compute the standard errors associated with the maximum likelihood estimators, similar to Giampieri et al.'s proposition (2005), the Monte Carlo method can be used to calculate the covariance matrix for parameters. All the algorithm codes are written in Matlab and can be presented if required.

In this paper, the four parameters concerned are a_{11}, a_{22}, p and K. For each set of data, firstly, the estimated HMM parameters are used to generate 100 series of observations (number of re-samples), each the same length as the original data. Secondly, for each generated sequence, the EM algorithm is used to re-estimate the parameters. The covariance matrix C is computed following the formula below:

$$C = \frac{1}{100-1} \sum_{j=1}^{100} (V_j - \overline{V})' \cdot (V_j - \overline{V}), \tag{6}$$

where $\overline{V} = \frac{1}{100} \sum_{j=1}^{100} V_j$ and V_j is a vector storing four parameters generated by EM algorithm each time. Hence, the standard error of each parameter derives from the square root of the elements on the diagonal.

3. SIMULATION RESULTS

Two series of data were simulated to show how EM algorithm works. In this simulation, each series contains 100 observations and the algorithm is performed twice, using two sets of initial values respectively. It is clear that the algorithm gives satisfactory results by both sets of initial values, even though the results from the second set of initial values show higher standard errors as the second set differs significantly from true parameters.

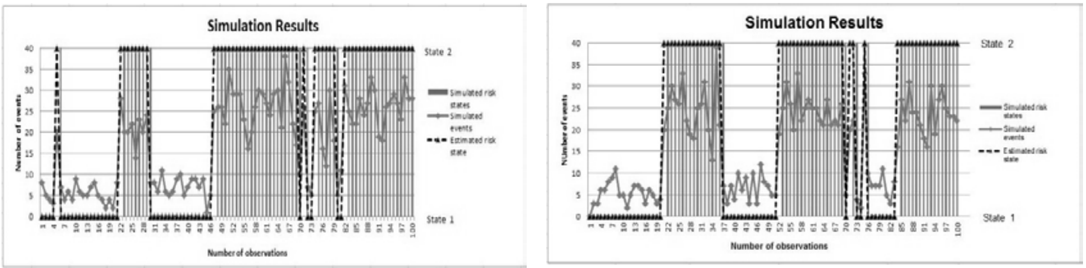
Table 1: Simulation Results

Parameters	True Parameter	Initial Value (1^{st} set)	Estimated result		Initial Value (2^{nd} set)	Estimated result
a11	0.9	0.7	0.9311(0.0372)		0.1	0.8910 (0.0500)
a22	0.9	0.7	0.9069 (0.0540)		0.1	0.9243(0.0493)
p	0.006	0.001	0.0057 (0.0003)		0.001	0.0059 (0.0004)
K	4	2	4.2519 (0.2503)		2	4.0880 (0.2896)

This table shows the simulation results using two sets of initial values listed in the 3rd and 5th column. The column 'True Parameter' lists the parameters used in the simulation; column 'Estimated Result' shows the parameters estimated by EM algorithm. Standard errors are given in parenthesis and the computation process is describe in the section 'METHODOLOGY'

Figure 1 below plots the simulated risk states along with the estimated risk states (generated by Viterbi algorithm). Obviously, the algorithm is capable of detecting hidden risk switching time points with a high accuracy.

Figure 1: Simulated Risk States versus Estimated Risk States



These two sub-figures show the simulated risk states and estimated risk. The left one uses the 1st set of initial values while the right one uses the other set. The solid line indicates the simulated number of events; the dashed line shows the estimated risk states while the shaded bars indicate simulated risk states.

4. DATA

4.1 Sources

In this paper, four sets of data are extracted from the appendices in Standard & Poor's and Moody's annual research reports. The statistics are collected annually and they report the changes in default counts from the first day to the last day of each year. Below is a detailed description of the data:

Global corporate issuer default counts: The data covers the period of 1920 to 2011. It is extracted from Moody's annual default study (2012) and the statistics in Exhibit 16 only covers Moody's rated corporate issuers.

Asia-Pacific corporate issuer default counts: The data covers the period of 1990 to 2011 - the modern era of bond issuance in this area. Specifically, the geographic regions concerned are East Asia (excluding Japan), India, Australia and New Zealand. It is extracted from Moody's Default and Recovery Rates of Asia-Pacific Corporate Bond and Loan Issuers, Exhibit 9 (2012).

U.S corporate issuer default counts: The data covers the period of 1981 to 2011. The statistics are acquired from all financial and non-financial entities. It is extracted from Standard & Poor's annual U.S. corporate default study, Table 6 (2012).

European corporate issuer default counts: The data covers the period of 1981 to 2011.

The geographic regions concerned are listed in Standard & Poor's Appendix I (2012).

The dates of business cycle expansions and contractions are plotted in Figure 2c, Figure 2d and Figure 3. The periods are announced by NBER and CEPR's dating committee (2012).

4.2 Default Definition

This paper follows Moody's and Standard & Poor's definitions of default, which are shown in each annual research report (See S&P's Annual European corporate study, Appendix I (2012)).

5. ESTIMATION RESULTS

5.1 Parameter Estimation Results

To estimate the parameters in the HMM, actual default data, as described in the data section, constitutes the observation sequence. Within each year, the probabilities that a bond would remain in either 'normal risk state' or 'enhanced risk state' are estimated by EM algorithm. P is the average default rate while K accounts for the 'infectious effect'. All results are shown in Table 2, where standard errors are presented in parentheses:

Table 2: Parameter Estimation Results

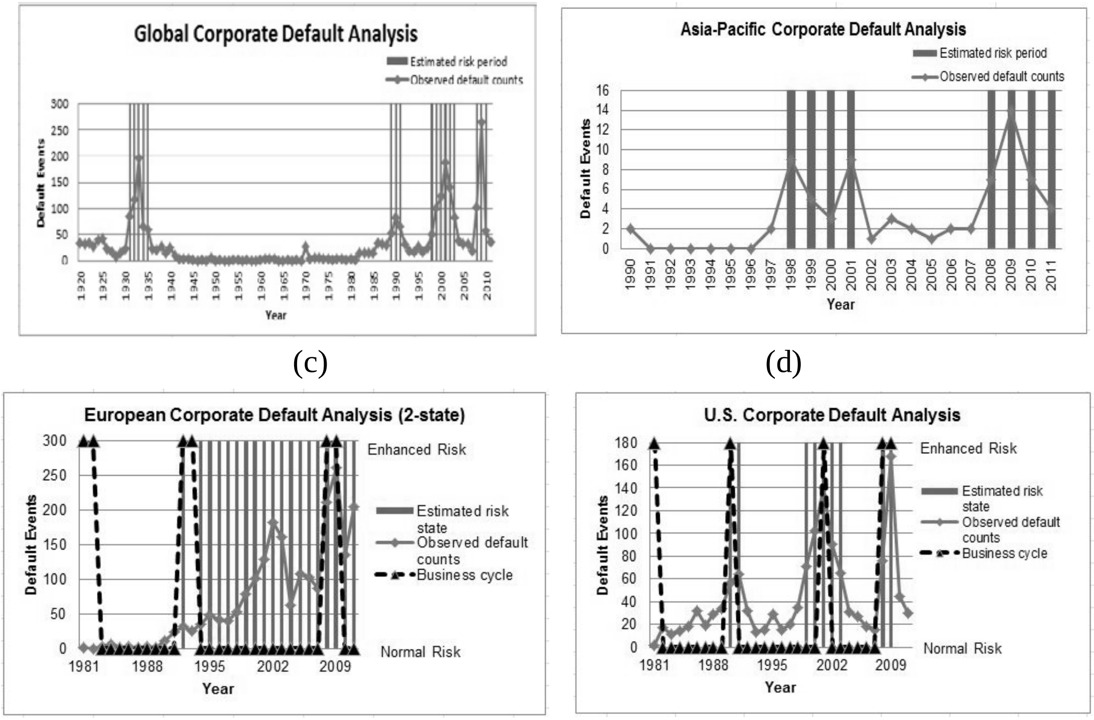
Region / Parameter	Globe	Asia-Pacific	Europe	the U.S.
a_{11}	0.9459 (0.0285)	0.8491(0.2336)	0.9444 (0.3324)	0.8571 (0.0800)
a_{22}	0.7647 (0.1812)	0.8458 (0.1657)	0.9999 (0.0743)	0.6667 (0.2061)
p	0.0007(0.0001)	0.0001(0.0004)	0.0002(0.0038)	0.0006(0.0000)
K	8.95 (0.5463)	6.66 (1.1012)	7.90 (1.2468)	4.06 (0.2365)

This table displays the parameters estimated using data from the Globe, Asia-Pacific area, Europe and the U.S. Detailed descriptions of the data source are covered in the section 'DATA'

Along with the estimated parameters, the most probable hidden state path can be computed using the Viterbi algorithm. Sub-figures in Figure 2 below show the estimated risk-switching process. For Europe and the U.S., business cycles are plotted on the graph, implying the actual risk states of the economy. The dates of business cycles are extracted from official dating organizations - the NBER in the U.S. and the CPER in Europe (2012).

Figure 2: Estimated Risk-switching Process Along With Business Cycles

(a) (b)



These figures illustrate the estimated risk-switching time points generated from EM algorithm. In all graphs, an appearance of a shaded bar shows that the macro-economy stays at 'Enhanced Risk' state; otherwise, it remains at the 'Normal Risk' state. The dashed line represents the risk levels indicated by business cycles; the solid line plots the observed default counts, using geographic data described in the section 'DATA'.

5.2 Analysis

Generally, our model can efficiently detect the macro-economic risk level since the hidden states discovered by the HMM coincide with the actual situation. The estimated hidden state paths in the above Figures 2 (a, b, c, d) reflect all the periods of financial downturn and the underlying causes of the switch in risk states are discussed below.

To be specific, in 1932, our model diagnosed a switch in risk states – from normal risk state to enhanced risk state (See Figure 2a). This resulted from the 'Great Depression' in 1929, when world trade and Europe's credit structure collapsed. In 1932, the U.S. economy declined severely and the disaster spread to Europe and Asia-Pacific countries such as Thailand and Australia. The financial environment remained unfavorable to investors until 1935, when the U.S. government intervened and the global labor union regained its power. This is when the risk state detected by our model switches back to 'normal risk state'.

During 1989-1991, Japan's real estate market underwent a catastrophic period. Though Japan is not included in any of the geographic regions analyzed above, it is a developed country and hence has large amounts of international trade across the globe. The crash of its housing market led to high corporate bond default rates and therefore, the global investors were forced to bear the financial loss. This period is again discovered by our model(See Figure 2a for the global data and Figure 2c and 2d for results from individual analyses).

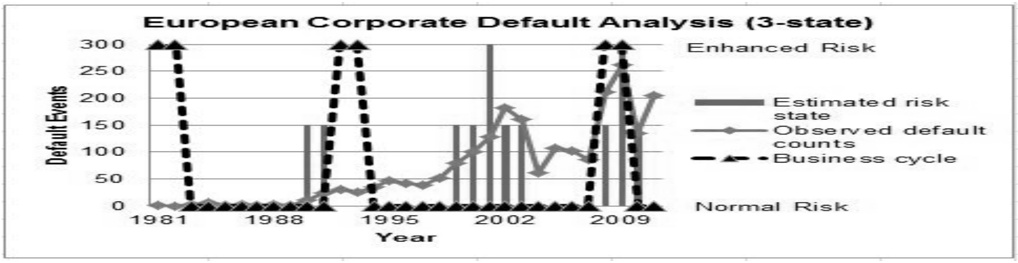
Ten years later, as illustrated in all four figures, 2001 was the year when recession began. After a boom in the stock market, the Nasdaq collapsed in 2001 due to the burst of the dotcom bubble. As a result, a significant amount of corporate bonds defaulted and most technology companies filed for bankruptcy (See Figure 2d, a switch in states happens in 1999).

Shown in all four subfigures, 2008 was the year when the subprime mortgage crisis triggered the global recession and it is when the estimated macro-economic risk state switches to ‘enhanced risk state’. This also induced the European sovereign debt crisis since Spain and Greece were not able to refinance their government debt. Though HMM detects most of the risk-switching time points in the macro-economy, it does not account for all economic situations. Taking the Asia-Pacific Analysis as an example, HMM fails to detect the enhanced risk period in 1990 (See Figure 2b) due to the lack of data. In this case, we only focus on the modern debt era, namely starting from 1990. This may explain the incompleteness of the estimated hidden state path.

Another surprising result worth mentioning here is that the hidden state discovered by our model does not fit well with the actual business cycle in the European Corporate Default Analysis (See Figure 2c). One possible reason might be that in 1993, when the European Union was established, business relationships between its member countries became closer. As was listed in S&P’s annual report, the number of new issuers and the corresponding defaults each year increased dramatically after 1993 (Vazza & Torres, 2012). This is when our model detects a switch in risk states. Gradually, the intrinsic country-specific economic conditions of EU members converge, which are no longer consistent with the historical conditions over the time period analyzed. This, to some extent, explains the inability of our model to discover the accurate hidden state sequence.

We attempted to address the problem mentioned above by designing a 3-state HMM, adding a ‘middle risk state’ to the original model.

Figure 3: 3-state HMM for European Corporate Default Analysis



In this figure, the highest shaded bar indicates ‘Enhanced Risk’ state, the bar of half its length represents 'Middle Risk’ state while the years without a bar imply that the macro-economy remain at ‘Normal Risk’ state.

Compared with the 2-state HMM, adding one more state produces better results since the previously missed switches are detected successfully. In terms of the Chi-square goodness-of-fit test, the calculated p-value for the 3-state HMM is 0.0012 (See Table 3), indicating that binomial distribution assumption is marginally significant at a 1% significance level in this case.

6. CHI-SQUARE GOODNESS-OF-FIT TEST

Following the procedures proposed by Sullivan (2010), a Chi-Square goodness-of-fit test can be conducted as follows to measure the appropriateness of the distribution assumption in this model. For each geographic region, we conducted this test on either ‘Enhanced Risk State’ data or ‘Normal Risk State’ data, choosing the one with a longer observation sequence. Results drawn from four independent tests are listed in Table 3. To ensure the validity of the test, regrouping is applied so that more than 80% of the expected frequencies of each group are larger than five.

Table 3: Chi-Square Goodness-of-fit Test

Region / Parameter	Globe	Asia-Pacific	Europe (2-state)	Europe (3-state)	the U.S.
Number of groups	4	2	3	3	4
χ^2	5.083	1.167	41.167	13.396	5.500
Degree of freedom	3	1	2	2	3
p-value	0.1658	0.2800	0.0000	0.0012	0.1386

This table displays the results from the Chi-square goodness-of-fit test (Detailed analysis see APPENDIX) .

According to Table 3, at a 5% significance level, the binomial distribution assumption in most cases (excluding Europe) is not rejected. Clearly, in the 3-state HMM, the test statistic in Europe’s case falls substantially. Though it is not significant at a 5% significance level, it is marginally significant at a 1% significance level. As expected, in most of the geographic regions, the hypothesized binomial distribution is valid. However, in terms of the corporate default data from Europe, our model is less competent in identifying the actual risk switching process. It is possible that the business cycle itself is not a flawless indicator of the macro-economic risk level.

7. FURTHER DISCUSSION

The model implemented in this paper is a 2-state hidden Markov model. Though in most cases it captures the macro-economic risk switching process, it still needs further improvements. One major limitation of this model lies in the lack of training data, which may lead to the inaccuracy of the results produced. Secondly, in this paper, we simply follow BET's two-state proposition. However, suggested by Knoblauch (2004), heuristics can be applied to optimize the number of states needed in HMM. Further, in this model, we assume that annual observations are statistically independent, but in the real world, annual default events may correlate with each other. More generalized models should therefore incorporate correlation effects into the original model. Additionally, we only use one observation sequence in this model and it is possible that a multi-sequence HMM would be more effective in explaining data. Further research can also focus on other influential factors in determining macro-economic risk states such as the one implied in this paper, the supply risk (due to a surge in the number of new issuers).

8. CONCLUSION

In this paper, a hidden Markov model is applied to explain the corporate default data in four geographic regions. To estimate the hidden macro-economic risk states, the Viterbi algorithm is used to find the most probable state switching path. In most cases, the estimated risk switching processes are in accord with those indicated by real business cycles. In addition, the goodness-of-fit test statistically shows the validity of the binomial distribution assumption. In Europe's case, however, the two-state HMM lacks explanatory power. After extending it to a 3-state model, it is clear that the detected risk switching process fits better with the actual fluctuations in the macro-economy.

APPENDIX

Table 1: Chi-Square goodness-of-fit test for Global corporate default data

Groups	Real Frequency	Expected Frequency	Component Chi-square
0-10	41	47	0.765957447
11-21	12	8	2
22-32	14	15	0.066666667
>33	7	4	2.25

Sum of Chi-square	5.082624113

Table 2: Chi-Square goodness-of-fit test for Asia-Pacific corporate default data

Groups	Real Frequency	Expected Frequency	Component Chi-square
0	6	8	0.5
>1	8	6	0.666667
Sum of Chi-square			1.166667

Table 3: Chi-Square goodness-of-fit test for European corporate default data (2-state model)

Groups	Real Frequency	Expected Frequency	Component Chi-square
0-10	0	2	2
11-16	0	11	11
>16	19	6	28.16667
Sum of Chi-square			41.16667

Table 4: Chi-Square goodness-of-fit test for European corporate default data (3-state model)

Groups	Real Frequency	Expected Frequency	Component Chi-square
0-5	9	16	3.0625
5-40	4	2	2
>40	8	3	8.333333
Sum of Chi-square			13.39583

Table 5: Chi-Square goodness-of-fit test for U.S. corporate default data

Groups	Real Frequency	Expected Frequency	Component Chi-square
0-6	1	2	0.5
6-18	9	5	3.2
18-30	6	10	1.6
>30	6	5	0.2
Sum of Chi-square			5.5

REFERENCES

[1] Center for economic policy research (2012) *Euro Area business cycle dating committee.* Retrieved August 6, 2012, from: http://www.cepr.org/data/dating/

[2] Davis, M. and Lo, V. (2001) Modeling default correlation in bond portfolios *In: Mastering Risk, Voloume 2: Applications.* London: Pearson Education, p.141-151

[3] Knoblauch, D. (2004) Data Driven Number-of-States Selection in HMM Topologies: *Eighth International Conference on Spoken Language Processing, 4-8 October, 2004 Jeju Island, Korea,*

[4] Gilchrist, S., Ortiz,A. and Zakarajsek, E. (2009) Credit Risk and the Macroeconomy: Evidence from an Estimated DSGE Model: *FRB/JMCB conference "Financial Markets and Monetary Policy," 4-5 June, 2009 Washington D.C.*

[5] Giampieri, G. , Davis, M. and Crowder M. (2005) A hidden markov model of default interaction, *Quantitative Finance*, 5, p. 27-34.

[6] Moody's Investors Service (2012) *Annual Default Study: Corporate Default and Recovery Rates, 1920-2011* New York: Moody's, 140015

[7] Moody's Investors Service (1996) *The Binomial Expansion Method Applied to CBO/CLO Analysis* New York: Moody's, 10007

[8] Moody's Investors Service (2012) *Default and Recovery Rates of Asia-Pacific Corporate Bond and Loan Issuers, Excluding Japan, 1990 -1H2011* New York: Moody's

[9] National Bureau of Economic Reserach (2012) *U.S. business cycle expansion and contractions.* Retrieved August 8, 2012, from: http://www.nber.org/cycles.html

[10] Rabiner, L. (1989). *A tutorial on hidden Markov models and selected applications in speech recognition,* Proceedings of the IEEE, vol. 77 p. 257-286.

[11] Sullivan, M. (2010) *Fundamentals of Statistics 3*rd ed. New Jersey: Prentice Hall.

[12] Vazza, D., Kraemer, N. and Gunter, E. (2012) *2011 Annual U.S. Corporate Default Study And Rating Transitions.* Retrieved August 5, 2012, from: http://www.standardandpoors.com/ratings/articles/en/us/?articleType=HTML&assetID=1245331026864

[13] Vazza, D. and Torres, J. (2012) *2011 Annual European Corporate Default Study And Rating Transitions*. Retrieved August 8, 2012, from: http://www.standardandpoors.com/ratings/articles/en/us/?articleType=HTML&assetID=1245331551763