GARCH Models for Inflation Volatility in Oman

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ABSTRACT

Monthly inflation time series of Oman were taken to compare the ability of a number of specifications of Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models in mean to capture and describe the persistence of conditional variance. The standard GARCH (s-GARCH) and exponential GARCH (e-GARCH) models with Autoregressive Moving Average (ARMA) and Autoregressive Fractionally Integrated Moving Average (ARFIMA) means were considered. The skew generalized error distribution as conditional distribution of innovation was assumed because of the shape of the empirical distribution of the series. The ARMA identification was made by sample autocorrelation (SAC) and partial sample autocorrelation (PSAC) while the fractional differencing parameter of ARFIMA was estimated from the observed time series. These specifications with both the s-GARCH and e-GARCH were compared for their adequacy based on the standard errors of the parameter estimates, mean square error, Log Likelihood, Modified Box-Pierce Chi-Square, information statistics, goodness of fit of the assumed conditional density. Based on these criterion, it was observed that e-GARCH specifications clearly performs better than s-GARCH while the ARIFMA model fits better than ARMA in both specifications indicating that long memory models may be best suited for forecasting inflation volatility in Oman as was the case for the United States. More over the conditional variance was better represented by e-GARCH rather than s-GARCH model. It was, therefore, suggested that e-GARCH (1,1)-ARFIMA(1,d,1) model with skewed generalized error distribution of residuals should be preferred for short term forecasting.

Key words: Time series, GARCH Models, Inflation, Conditional Variance, Forecasting.

1. INTRODUCTION:

GARCH models are widely used for forecasting volatility in the financial and economic time series in general and particularly for modeling conditional dynamics of inflation uncertainties. Inflation dynamics might significantly effect the investment decisions and purchasing power subsequently leading to distortion in stability of the price mechanism. This has been a matter of great concern for the economists and policy makers compelling them to consider inflation movements as one of the prime objectives in economic and financial analysis and research. Since the seminal work of Engle (1) on the basic autoregressive conditional heteroskedasticity (ARCH) model for the U. K. inflation and the subsequent generalization by Bollerslev (2) to the generalized ARCH (GARCH) specification, various modifications of the basic GARCH model have been proposed. There has been considerable interest in combining Box-Jenkins ARMA regime with the GARCH assuming skewed innovation distributions to examine the relationship between Inflation and Inflation uncertainty e.g. Alexander and Lazar (3), Haas *et al*.(4). More recently, it is being argued that the inflation data is characterized by the long memory persistence e. g. Cheung and Chung(5), therefore Auto Regressive Fractionally Integrated Moving Average (ARFIMA) models might better describe inflation uncertainty. In the present study we attempt to investigate the performance of this approach to capture the volatility persistence in Inflation time series of Oman.

2. MODEL DESCRIPTION:

The classical representation of the generalized ARCH model, GARCH (p, q) model, is specified by the equation : $Y_t = f(X_t, \delta) + \xi_t$ where $\xi_t | \psi_{t-1}$ has $D(0, \sigma^2_t)$ and $f(X_t, \delta)$ is the conditional mean and is matrix of explanatory variables while δ is a vector of parameters. The error term ξ_t is assumed to have skew generalized error distribution and is conditional on information available till point of time t-1 i.e. ψ_{t-1}. This specification allows us to define dynamics for the conditional mean from the general ARMA model with the addition of ARCH-in-mean effects introduced in Engle(6). If the inflation be generated by a stochastic process $\{Y_t\}$ which can be described by ARIMA(p,d,q) process as: $\Phi(B)(1-B)^d(Yt) = \Theta(B)\,\xi_t$ where, B is back shift operator, $\Phi(B) = 1-\varphi_1 B-,\ldots,-\varphi p B^p$, $\Theta(B) = 1-\theta_1 B- -\theta q B^q$ are polynomials in B; φi ($i = 1,\ldots, p$) and θi ($i = 1,\ldots,q$) are the autoregressive and moving average parameters respectively. If $\{Yt\}$ is an integrated series $I(d)$ for d=1,2,3,..,k we term such series a 'short range' dependence process which can be modeled using the Autoregressive Integrated Moving Average (ARIMA(p,d,q)) . However, if d is not an integer i.e d <1, we term the series 'long range' dependence process or fractionally integrated series of order d i.e. Autoregressive Fractionally Integrated Moving Average ARFIMA(p,d,q). The overall pattern of the fractionally integrated series is characterized by a sequence of 'long swings' around a fairly stable average. Such pattern is interpreted as long memory in which temporal dependence falls off very slowly over time Mills(7).

Standard and exponential GARCH specifications for the volatility of the error structure denoted by s-GARCH and e-GARCH are compared in this study. The s-GARCH specification is given by $\sigma^2_\tau = \omega + \sum_\iota \alpha_\iota \xi^2_{t-\iota} + \sum_\iota \beta_\iota \sigma^2_{\tau-\iota}$ and that of the e-GARCH as

$$\mathrm{Log}(\sigma^2_t) = \omega + \sum_\iota (\alpha_\iota Z_{\tau-\iota} + \gamma_\iota (|Z_{\tau-\iota}| - E|Z_{\tau-\iota}|) + \sum_\iota \beta_\iota \mathrm{Log}(\sigma^2_{t-\iota})$$

Where ω is intercept term, Z is standardized innovations, γ is size effect and the persistence of shocks to volatility is given by $\alpha_i+\beta_j$. The model was fitted using rugarch (8) which uses maximum likelihood method of estimation.

3. DATA AND ANALYSIS:

Monthly data of consumer's price index (CPI) of Oman was obtained from the web site of Ministry of National Economy (9) over a period of January 2001 to September 2011 comprising 129 data values. The inflation was measured as 100 times the first differences of the logarithms of the CPI. Figure 1 presents a plot of the inflation time series which reveals a considerable degree of volatility clustering and the series clearly appears non stationary. In fact there is a structural break and shift in the trend at around 2008 which might be due to high levels of volatilities in the crude oil prices and the fact that Oman's economy heavily depends on crude oil production. Table 1 gives the preliminary analysis indicating markedly high skewness and kurtosis suggesting a fat tailed underlying conditional distribution which is far from normal. The box plot shows that the right tail of the distribution of inflation is very long and median is significantly smaller than mean indicating the clustering of volatility towards higher values. Therefore it seems very appropriate to assume the skew generalized error distribution as conditional distribution of innovation. This also give the indication that higher inflation causes higher uncertainty.

Table 2 presents the results of various information statistic for the adequacy of fitting both e-GARCH and s-GARCH with each of ARMA and ARFIMA specifications. It is clearly evident from this table that all the five criterion i.e. –log-likelihood, Akaike, Bayes

Bayes, Shibata, and Hannan-Quinn have smaller values for e-GARCH as compared to s-GARCH specification for variance of the innovations. This leads us to conclude with a reasonable certainty that volatility of inflation in Oman can best be modeled with e-GARCH(1,1).

Figure 1

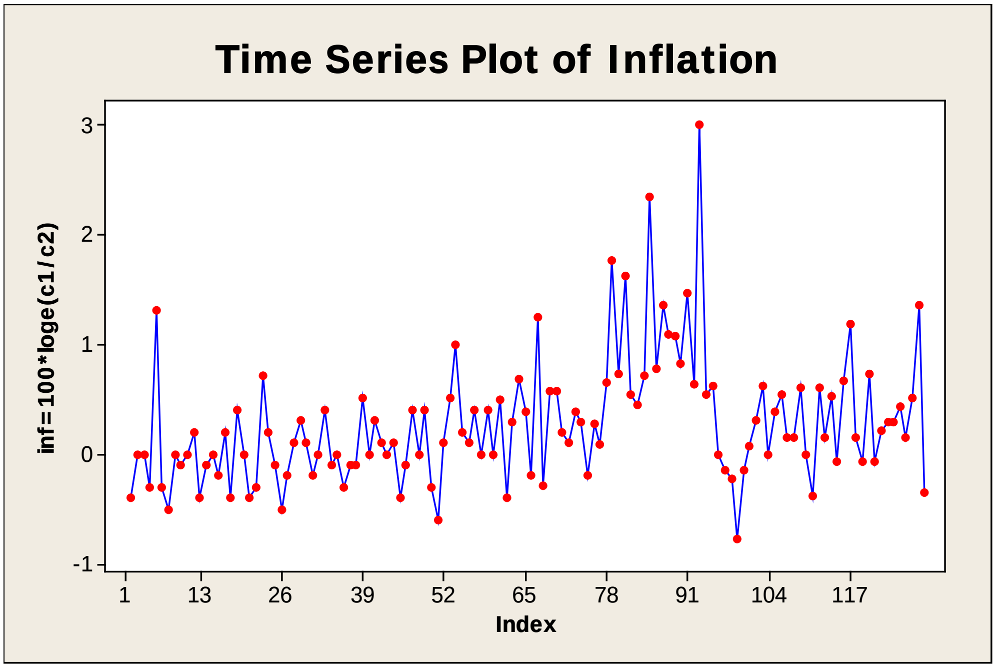

Table 1:

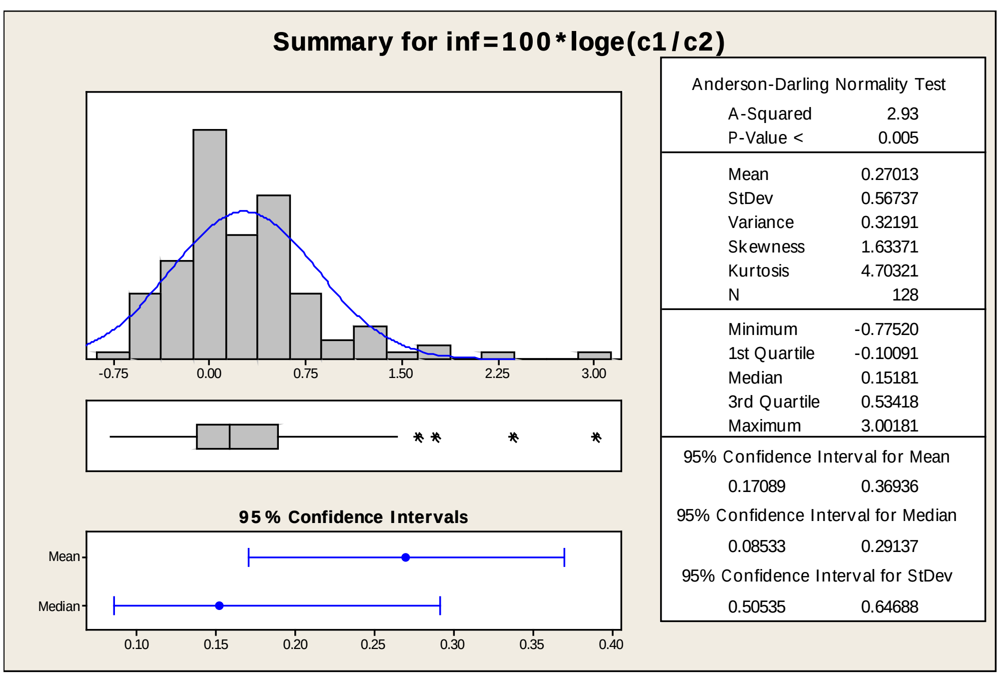

Table 2. Information criterion for adequacy of the specified models

	e-GARCH(1,1)		s-GARCH(1,1)	
	ARMA(1,0,1)	ARFIMA(1,d,1)	ARMA(1,0,1)	ARFIMA(1,d,1)
-log-likelihood	65.18787	64.1664	-74.43438	-72.33211
Akaike	1.2469	1.2465	1.3855	1.3669
Bayes	1.4570	1.4800	1.5723	1.5771
Shibata	1.2365	1.2338	1.3771	1.3565
Hannan-Quinn	1.3322	1.3413	1.4613	1.4523

On the other hand the mean of inflation process represented by ARFIMA has smaller goodness of fit values except for Bayes criteria. Thus we can conclude that ARFIMA(1,1) can better represent the mean inflation suggesting that the inflation in Oman has a long memory persistence . Therefore e-GARCH(1,1)-ARFIMA(1,1) model is seen as having ability to describe long memory dynamics of Oman inflation and inflation uncertainty. Tables 3 and 4 give the parameter estimates, their stander errors and t-values of the proposed models. The parameter estimates of all the selected models are highly significat as shown by the higher t-values indicating that the inflation data are quite persistent and display strong GARCH effect and have a long memory. The fraction parameter estimates are significant under each GARCH specification and are less than .5 implying considerable long term persistence in the Oman inflation data. This is consistent with the results reported by the previous studies and support the Friedman–Ball hypothesis that inflation increases the inflation uncertainty as investigated by the empirical studies on this subject [10,11, 12].

Table 3: Estimates of the parameters of ARMIMA with exponential and standard GARCH specifications.

e-GARCH(1,1)-ARFIMA(1,d,1)				s-GARCH(1,1)-ARFIMA(1,d,1)			
Parameter	Estimate	Std. Error	t value	Parameters	Estimate	Std. Error	t value Pr(>|t|)
mu	0.294813	0.049732	5.92798	mu	0.397321	0.000853	465.8042
ar1	0.729517	0.100245	7.27736	ar1	0.955547	0.003672	260.2209
ma1	-0.527876	0.026530	-19.89768	ma1	-0.143069	0.001143	-125.1830

arfima	-0.015132	0.002352	-6.43296	arfima	-0.684266	0.002384	-287.0062
omega	-0.345298	0.080107	-4.31046	omega	0.018832	0.005277	3.5685
alpha1	0.338678	0.128808	2.62932	alpha1	0.017536	0.000480	36.5695
beta1	0.782181	0.000220	3561.67603	beta1	0.905358	0.001306	693.0292
gamma1	-0.726074	0.010720	-67.72916				
skew	1.334849	1.422184	0.93859	skew	1.467859	0.001088	1349.5874
shape	1.890572	0.320957	5.89042	shape	1.114718	0.336733	3.3104

Table 4: Estimates of the parameters of ARMA with exponential and standard GARCH specifications.

e-GARCH(1,1)-ARFIMA(1,0,1)				s-GARCH(1,1)-ARFIMA(1,0,1)			
Parameter	Estimate	Std. Error	t value	Parameters	Estimate	Std. Error	t value
mu	0.30091	0.008361	35.9896	mu	0.308677	0.005853	52.7409
ar1	0.74282	0.018771	39.5733	ar1	0.886088	0.008411	105.3431
ma1	-0.58239	0.084567	-6.8867	ma1	-0.715141	0.001095	-653.2902
omega	-0.28450	0.006880	-41.3496	omega	0.016809	0.004026	4.1756
alpha1	0.26320	0.053311	4.9370	alpha1	0.039239	0.018524	2.1183
beta1	0.82387	0.000656	1255.6456	beta1	0.891669	0.000511	1743.4617
gamma1	-0.59854	0.023685	-25.2703				
skew	1.50897	0.149620	10.0853	skew	1.354319	0.013254	102.1816
shape	2.06167	0.145562	14.1635	shape	1.111084	0.104693	10.6128

4. CONCLUSION:

Various model specifications to describe the volatility in mean and variance are compared for their performance to fit Oman inflation time series data in terms of five different information criterions for the model adequacy. The empirical evidence suggests that the inflation data reveals a long memory persistence in mean as represented by ARFIMA(1,1) component of the model while the conditional volatility is driven by e-GARCH(1,1) process. These proposed specification for the level and volatility compares, indeed, much better than the alternative ARMA and s-GARCH. More over the asymmetric innovations with a very high skew and kurtosis are found to be well captured by the skew generalized error distribution with very high p-values.

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