Review of Integrative Business & Economics Research

Heterogeneity in Wage Rigidity and Monetary Policy

Muneya Matsui
Department of Business Administration, Nanzan University, 18 Yamazato-cho Showa-ku Nagoya-shi Aichi-ken 466-8673, Japan. E-mail: mmuneya@nanzan-u.ac.jp.

Taiyo Yoshimi
Corresponding author. Department of Economics, Nanzan University, 18 Yamazato-cho Showa-ku Nagoya-shi Aichi-ken 466-8673, Japan. E-mail: yoshimi@ic.nanzan-u.ac.jp.

Abstract

We construct a DSGE model with heterogeneity in wage rigidity, and compare the welfare effects of several monetary policy rules, including the standard Taylor rule. We find that monetary policy rules considering wage inflations improve the welfare by stabilizing employment against the labor productivity shock. Further, a monetary policy rule with consideration for the inflation of wages which are adjusted more frequently, namely more flexible wage inflation, becomes better than one with consideration for less flexible wage inflation. It realizes smaller welfare loss than the standard Taylor rule when the weight on the labor disutility in the household's utility function is sufficiently large.

Keywords: Heterogeneity in Wage Rigidity, Search and Matching, Monetary Policy

1. Introduction

Nominal wage rigidity has always attracted macro-economists, since it has the potential to be the most essential source of real effects on the economy when nominal macroeconomic shocks occur. For instance, researchers including Erceg et al. (2000) and Galí (2012) have constructed models with wage rigidity, and studied desirable monetary-policy implementations based on the macroeconomic dynamics. The same is true for empirical studies. Following the seminal work of Kahn (1997), many researchers have tried to characterize the wage rigidity empirically, and some of them have pointed out the existence of its heterogeneity among both workers and firms. Kahn (1997) focused on downward wage rigidity in the U.S. economy[1] and found that wage and salary workers suffer wage cuts with different frequencies. With the same database, Christofides and Stengos (2007) implies that those differences can also be found among wage workers. Knoppik and Beissinger (2013) found that workers with less job experience tend to face wage cuts and renegotiations more often than workers with more experience. This implies more wage flexibility for the former workers. Devicienti et al. (2007) investigated wage rigidity in the Italy dataset[2], and found that wage rigidity differs across industries. They also found more frequent wage rises for male workers than female workers. All of these empirical findings imply the existence of the heterogeneity in wage rigidity in reality. However, there are few theoretical papers which focus on this empirical aspect. In this research, we try to fill this gap by introducing a simple heterogeneity of wage rigidity into a standard DSGE model and we consider welfare implications of alternative monetary policy rules.

We also introduce the search and matching friction into our model, and consider unemployment dynamics. As standard New Keynesian models have been deemed incapable of explaining unemployment, many recent papers have dealt with it to better approach reality. The technique originates in seminal works by Diamond (1982), Mortensen (1982), and Pissarides (1985). Merz (1995) and Andolfatto (1996) applied the standard search theory to an RBC model[3]. Hall (2005) and Shimer (2005) added real wage rigidity to the labor market frictions; this was further extended in Gertler and Trigari (2009) to satisfy the standards of the RBC model. Zanetti (2007) also deals with unemployment with regard to labor unions. See also Thomas (2008), who, importantly, uses a staggered nominal wage rather than a real one. More generally, Blanchard and Galí (2010) constructed a model with labor market frictions, real wage rigidities, and a staggered price setting. Our main framework is based on Gertler et al. (2008), though ours has a dual labor market and is capable of analyzing the heterogeneity in wage rigidity[4].

This paper is structured in the following manner. The following section presents the basic settings of the model. In Section 3, we analyze welfare implications in terms of the second moments of variance involved. Section 4 briefly discusses the remaining issues and concludes the paper.

---

[1] The database of Panel Study of Income Dynamics, as known as PSID was studied.
[2] Worker History Italian Panel (WHIP).
[3] See the excellent summary by Pissarides (2000).
[4] Mattesini and Rossi (2009) also consider dual labor market for Walrasian and unionized workers, although their focus is not on heterogeneous wage rigidity.

2. Model

2.1. Representative Household

There are two types of workers: type-1 and type-2. Since each family is assumed to have both types of workers and jobless persons, it provides perfect consumption insurance among them, i.e. consumption amount is the same for all members. As usual, workers lose utility from employment. The household chooses consumption c_t, government bonds B_{t+1}, capital utilization v_t, investment i_t, and physical capital k_t^p, so as to maximize its utility

$$E_t \sum_{s=0}^{\infty} \beta^s W(c_{t+s}, n_{1,t+s}, n_{2,t+s}), \tag{2.1}$$

where

$$W(c_t, n_{1,t}, n_{2,t}) \equiv \ln c_t - \Omega \frac{n_{1,t}^{1+\iota}}{1+\iota} - \Omega \frac{n_{2,t}^{1+\iota}}{1+\iota}. \tag{2.2}$$

Here Ω is the weight on the labor disutility and ι is the inverse Frisch elasticity of labor supply.

As we will describe, each wholesale firm i inputs utilized capital $k_t(i)$ and two types of workers, $n_{1,t}(i)$ and $n_{2,t}(i)$, for its production. Wholesale firms are continuously distributed on the unit interval. Labor market clearing implies

$$n_{\tau,t} = \int_0^1 n_{\tau,t}(i)di \quad (\tau = 1,2). \tag{2.3}$$

By normalizing the total population of household members to unity, we obtain the pool of unemployment workers searching for a job in period t,

$$u_t = 1 - n_{1,t-1} - n_{2,t-1}. \tag{2.4}$$

Let $w_{\tau,t}(i)$ be the real wage of type-τ workers $(\tau = 1,2)$, b_t unemployment benefit, r_t^k capital rental rate, Π_t lump sum profit, T_t lump sum transfer, p_t the nominal price level of the final good, r_t the nominal interest rate, and $A(v_t)$ the cost of capital utilization per unit of physical capital. Then the household's budget constraint is

$$\int_0^1 w_{1,t}(i)n_{1,t}(i)di + \int_0^1 w_{2,t}(i)n_{2,t}(i)di + \left(1 - n_{1,t} - n_{2,t}\right)b_t + r_t^k v_t k_{t-1}^p + \Pi_t + T_t - A(v_t)k_{t-1}^p + \frac{B_t}{p_t} = c_t + i_t + \frac{B_{t+1}}{p_t r_{t+1}}. \quad (2.5)$$

The household owns capital and chooses the capital utilization rate v_t which transforms physical capital into effective capital through

$$k_t = v_t k_{t-1}^p. \quad (2.6)$$

We assume

$$v_t = 1, \quad (2.7)$$

$$A(1) = 0, \quad (2.8)$$

$$\frac{A'(1)}{A''(1)} = \eta_v \quad (2.9)$$

in the steady state.

The physical capital accumulation equation is

$$(1-\delta)k_{t-1}^p + \left[1 - S\left(\frac{i_t}{i_{t-1}}\right)\right] i_t = k_t^p. \quad (2.10)$$

Here δ is the depreciation rate and $S(\cdot)$ is the adjustment cost of investment. In the steady state, we assume

$$S(1) = S'(1) = 0, \quad (2.11)$$

$$S''(1) = \eta_k > 0. \quad (2.12)$$

The first order conditions of household's optimization are

$$\lambda_t = \frac{1}{c_t}, \quad (2.13)$$

$$\lambda_t = r_{t+1}\beta E_t\left(\frac{\lambda_{t+1}p_t}{p_{t+1}}\right), \quad (2.14)$$

$$r_t^k = A'(v_t), \quad (2.15)$$

$$q_t^k\left[1 - S\left(\frac{i_t}{i_{t-1}}\right)\right] = q_t^k S'\left(\frac{i_t}{i_{t-1}}\right)\frac{i_t}{i_{t-1}} - \beta E_t q_{t+1}^k \frac{\lambda_{t+1}}{\lambda_t} S'\left(\frac{i_t}{i_{t-1}}\right)\left(\frac{i_t}{i_{t-1}}\right)^2 + 1, \quad (2.16)$$

$$q_t^k = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\{(1-\delta)q_{t+1}^k + r_{t+1}^k v_{t+1} - A(v_{t+1})\}. \tag{2.17}$$

Here q_t^k is the shadow price of capital and λ_t is the Lagrange multiplier on the budget constraint.

2.2. Unemployment, Vacancies and Matching

Recall that we assume that there are two types of workers indicated by type-1 and 2. Suppose that a member of the representative household is working as either type of employee. In the next period, he keeps his job with probability ρ, and is fired with probability $1-\rho$. Here ρ is an exogenous parameter, and the same for both types of workers. Once a worker is fired, he has to stay in the unemployment pool at least for one time period. And after that he will get a job as a type-1 employee with the probability $s_{1,t}$, and as type-2 with the probability $s_{2,t}$. Accordingly, the probability of staying jobless is $1-s_{1,t}-s_{2,t}$[5], whereas that of finding job is $s_{\tau,t}$, which is derived from the ratio of numbers of new hires or "matches", $m_{\tau,t}$, and unemployed people, u_t,

$$s_{\tau,t} = \frac{m_{\tau,t}}{u_t}. \tag{2.18}$$

We assume that $m_{\tau,t}$ follows the Cobb-Douglas matching function of unemployment and the total number of vacancies, $v_{\tau,t}$,

$$m_{\tau,t} = u_t^{\sigma} v_{\tau,t}^{1-\sigma}. \tag{2.19}$$

Here $v_{\tau,t}$ is defined by

$$v_{\tau,t} = \int_0^1 v_{\tau,t}(i)di, \tag{2.20}$$

where $v_{\tau,t}(i)$ is the number of vacancies of type- τ employment by each firm at time t. The probability a firm fills a vacancy, $q_{\tau,t}$, is given by

[5] In Appendix A, we prove that rates of hiring and dismissal of each type of employment are equalized in the steady state.

$$q_{\tau,t} = \frac{m_{\tau,t}}{v_{\tau,t}}. \tag{2.21}$$

2.3. Wholesale Firms

Each wholesale firm i produces output $y_t(i)$ following the technology function

$$y_t(i) = k_t(i)^{\alpha}[z_t n_t(i)]^{1-\alpha}, \tag{2.22}$$

where employment index $n_t(i)$ is defined by

$$n_t(i) = [n_{1,t}(i)]^{\omega_1}[n_{2,t}(i)]^{\omega_2}. \tag{2.23}$$

Here we assume $0 \leq \alpha \leq 1$, $\omega_1 + \omega_2 = 1$, $\omega_i \geq 0$, $i = 1,2$. Defining $\hat{X}_t$ as the deviation rate of respective variable X_t from its steady state, $\hat{z}_t$ is assumed to obey following AR(1) process:

$$\hat{z}_t = \rho^z \hat{z}_{t-1} + \zeta_t^z \tag{2.24}$$

where ζ_t^z is a white noise process with zero mean.

Combining the first-order conditions and the definition of $n_t(i)$, we obtain demand functions for each type of employee,

$$n_{\tau,t}(i) = \omega_\tau \frac{w_t(i)}{w_{\tau,t}(i)} n_t(i), \tag{2.25}$$

where the wage index is

$$w_t(i) = \left[\frac{w_{1,t}(i)}{\omega_1}\right]^{\omega_1}\left[\frac{w_{2,t}(i)}{\omega_2}\right]^{\omega_2}. \tag{2.26}$$

Thus,

$$w_t(i)n_t(i) = w_{1,t}(i)n_{1,t}(i) + w_{2,t}(i)n_{2,t}(i). \tag{2.27}$$

Aggregating both sides, we obtain

$$w_{1,t}n_{1,t} + w_{2,t}n_{2,t} = w_t n_t, \tag{2.28}$$

where wage indexes $w_{\tau,t}$ and w_t are defined so as to fulfill

$$w_{\tau,t}(i) = \int_0^1 w_{\tau,t}(i)\frac{n_{\tau,t}(i)}{n_{\tau,t}}di, \tag{2.29}$$

$$w_t = \int_0^1 w_t(i)\frac{n_t(i)}{n_t}di. \tag{2.30}$$

It is useful to define the hiring rate $x_{\tau,t}(i)$ as the ratio of new hires $q_{\tau,t}(i)v_{\tau,t}(i)$ to the existing workforce $n_{\tau,t-1}(i)$:

$$x_{\tau,t}(i) = \frac{q_{\tau,t}v_{\tau,t}}{n_{\tau,t-1}(i)}. \tag{2.31}$$

The total workforce is the sum of the number of surviving workers $\rho n_{\tau,t-1}(i)$ and new hires $x_{\tau,t}(i)n_{\tau,t-1}(i)$:

$$n_{\tau,t}(i) = (\rho + x_{\tau,t}(i))n_{\tau,t-1}(i). \tag{2.32}$$

Let p_t^w be the relative price of intermediate goods, $w_{\tau,t}^n$ be the nominal wage, r_t^k be the rental rate of capital, and $\beta E_t\Lambda_{t,t+1}$ be the firm's discount rate, where the parameter β is the household's subjective discount factor and where

$$\Lambda_{t,t+1} = \frac{\lambda_{t+1}}{\lambda_t}. \tag{2.33}$$

Then, the value of the firm $F_t(\cdot)$ is expressed as:

$$F_t\left(w_{1,t}^n(i), w_{2,t}^n(i), n_{1,t-1}(i), n_{2,t-1}(i)\right) = p_t^w y_t(i) - \frac{w_{1,t}^n(i)}{p_t}n_{1,t}(i) - \frac{w_{2,t}^n(i)}{p_t}n_{2,t}(i) -$$

$$\frac{\kappa}{2}\left[x_{1,t}(i)\right]^2 n_{1,t-1}(i) - \frac{\kappa}{2}\left[x_{2,t}(i)\right]^2 n_{2,t-1}(i) - r_t^k k_t(i) +$$

$$\beta E_t\Lambda_{t,t+1}F_{t+1}\left(w_{1,t+1}^n(i), w_{2,t+1}^n(i), n_{1,t}(i), n_{2,t}(i)\right). \tag{2.34}$$

Here we assume the costs for new hires, $\frac{\kappa}{2}\left[x_{\tau,t}(i)\right]^2 n_{\tau,t-1}(i)$. The first order condition for capital implies

$$r_t^k = p_t^w \alpha \frac{y_{it}}{k_{it}} = p_t^w \alpha \frac{y_t}{k_t}. \tag{2.35}$$

Firms choose $n_{\tau,t}(i)$ by setting $x_{\tau,t}(i)$ or, equivalently, $v_{\tau,t}(i)$. The firm's hiring decision yields:

$$\kappa x_{\tau,t}(i) = p_t^w a_{\tau,t}(i) - \frac{w_{\tau,t}^n(i)}{p_t} + \beta E_t \Lambda_{t,t+1} \frac{\partial F_{t+1}\left(w_{1,t+1}^n(i), w_{2,t+1}^n(i), n_{1,t}(i), n_{2,t}(i)\right)}{\partial n_{\tau,t}(i)}, \tag{2.36}$$

where marginal productivities are respectively defined by

$$a_t(i) \equiv (1-\alpha)\frac{y_t(i)}{n_t(i)} = (1-\alpha)\frac{y_t}{n_t} = a_t, \tag{2.37}$$

$$a_{\tau,t}(i) \equiv a_t \omega_\tau \frac{n_t(i)}{n_{\tau,t}(i)}. \tag{2.38}$$

By making use of the envelope theorem to obtain $\frac{\partial F_t(\cdot)}{\partial n_{\tau,t-1}(i)}$ and combining equations, we obtain

$$\kappa x_{\tau,t}(i) = p_t^w a_{\tau,t}(i) - \frac{w_{\tau,t}^n(i)}{p_t} + \beta E_t \Lambda_{t,t+1} \frac{\kappa}{2}\left[x_{\tau,t+1}(i)\right]^2 + \rho\beta E_t \Lambda_{t,t+1} \kappa x_{\tau,t+1}(i). \tag{2.39}$$

The hiring rate thus depends on the discounted stream of earnings and savings on adjustment costs.

For the wage bargaining, we define the values to the firm of having another worker at time t after adjustment costs are sunk, denoted by $J_{\tau,t}(\cdot)$. Differentiating $F_t(\cdot)$ with respect to $n_{\tau,t-1}(i)$, taking $x_{1,t}(i)$ and $x_{2,t}(i)$ as given yields:

$$J_{\tau,t}\left(w_{1,t}^n(i), w_{2,t}^n(i)\right) = p_t^w a_{\tau,t}(i) - \frac{w_{\tau,t}^n(i)}{p_t} + \beta E_t \Lambda_{t,t+1} \frac{\partial F_{t+1}\left(w_{1,t+1}^n, w_{2,t+1}^n, n_{1,t}, n_{2,t}\right)}{\partial n_{\tau,t}}. \tag{2.40}$$

By making use of the hiring rate condition and the relation for the evolution of the workforce, $J_{\tau,t}\left(w_{1,t}^{n}(i), w_{2,t}^{n}(i)\right)$ may be expressed as expected average profits per worker net of the first period adjustment costs, with the discount factor accounting for future changes in workforce size:

$$J_{\tau,t}\left(w_{1,t}^{n}(i), w_{2,t}^{n}(i)\right) = p_t^{w} a_{\tau,t}(i) - \frac{w_{\tau,t}^{n}(i)}{p_t} - E_t\Lambda_{t,t+1}\frac{\kappa}{2}\left[x_{\tau,t+1}(i)\right]^2 + \beta E_t\Lambda_{t,t+1}(\rho + x_{\tau,t+1}(i))J_{\tau,t+1}\left(w_{1,t+1}^{n}(i), w_{2,t+1}^{n}(i)\right). \tag{2.41}$$

2.4. Workers

In this subsection we develop an expression for a worker's surplus from employment. Let $V_{\tau,t}\left(w_{1,t}^{n}(i), w_{2,t}^{n}(i)\right)$ be the value of a worker employed at firm i and let U_t be the value of unemployment, i.e.

$$V_{\tau,t}\left(w_{1,t}^{n}(i), w_{2,t}^{n}(i)\right) = \frac{w_{\tau,t}^{n}(i)}{p_t} + \beta E_t\Lambda_{t,t+1}\left[\rho V_{\tau,t+1}\left(w_{1,t+1}^{n}(i), w_{2,t+1}^{n}(i)\right) + (1-\rho)U_{t+1}\right]. \tag{2.42}$$

These values are defined after hiring decisions at time t have been made and are in units of consumption goods. To construct the value of unemployment, we first define $V_{x,t}^{\tau}$ as the average value of employment conditional on being a new worker at t:

$$V_{x,t}^{\tau} = \int_0^1 V_{\tau,t}\left(w_{1,t}^{n}(i), w_{2,t}^{n}(i)\right)\frac{x_{\tau,t}(i)n_{\tau,t-1}(i)}{x_{\tau,t}n_{\tau,t-1}}di. \tag{2.43}$$

The value of unemployment U_t is expressed as

$$U_t = b_t + \beta E_t\Lambda_{t,t+1}\left[s_{1,t+1}V_{x,t+1}^{1} + s_{2,t+1}V_{x,t+1}^{2} + (1 - s_{1,t+1} - s_{2,t+1})U_{t+1}\right] \tag{2.44}$$

with

$$b_t = lk_t^{p}, \tag{2.45}$$

where l is a constant parameter. Note that the value of finding a job next period for an unemployed worker is $V^{\tau}_{x,t}$, the average value of working next period which is, namely, unemployed workers do not have a priori knowledge of which firms might be paying higher wages next period. Alternatively, they randomly flock to firms posting vacancies.

Given above relations, the net value of type- τ employment is given by

$$H_{\tau,t}\left(w^n_{1,t}(i), w^n_{2,t}(i)\right) = V_{\tau,t}\left(w^n_{1,t}(i), w^n_{2,t}(i)\right) - U_t. \tag{2.46}$$

Similarly, the average net value, $H^{\tau}_{x,t}$, is given by

$$H^{\tau}_{x,t} = V^{\tau}_{x,t} - U_t. \tag{2.47}$$

It follows that

$$H_{\tau,t}\left(w^n_{1,t}(i), w^n_{2,t}(i)\right) =$$

$$\frac{w^n_{\tau,t}(i)}{p_t} - b_t + \beta E_t \Lambda_{t,t+1}\left[\rho H_{\tau,t+1}\left(w^n_{1,t+1}(i), w^n_{2,t+1}(i)\right) - s_{1,t+1}H^1_{x,t+1} - s_{2,t+1}H^2_{x,t+1}\right]. \tag{2.48}$$

2.5. Nash Bargaining and Wage Dynamics

We introduce staggered Nash wage bargaining following Gertler et al.(2008), although we simplify the model by ignoring steady state trends of variables. Each period, a firm has a fixed probability $1-\lambda_\tau$ of wage negotiation. Thus, the coefficient λ_τ can be interpreted the degree of wage stickiness.

In our model, the fraction λ_τ of firms that cannot renegotiate their contract set their nominal wages at the last-period levels as

$$w^n_{\tau,t}(i) = w^n_{\tau,t-1}(i). \tag{2.49}$$

For the future use, we also define ratios of past and current nominal wage levels as

$$\pi^w_t \equiv \frac{w^n_t}{w^n_{t-1}}, \tag{2.50}$$

$$\pi^w_{\tau,t} \equiv \frac{w^n_{\tau,t}}{w^n_{\tau,t-1}}. \tag{2.51}$$

Once firms enter a new wage agreement, they negotiate with both existing and newly hired workers. The wage is chosen so that the firms and the marginal worker share the surplus from the marginal match. Given this, all workers belonging to the same type of employment contract, i.e. type-1 and 2, and the employed receive the same wage newly set through the negotiation. When firms and workers are not allowed to renegotiate the wage, workers receive the last-period nominal wage. We assume Nash bargaining, implying that the contract wage $w^n_{\tau,t}(i)$ is chosen to solve

$$\max_{w^n_{\tau,t}(i)}[H_{\tau,t}(w^n_{1,t}(i),w^n_{2,t}(i))]^{\eta}[J_{\tau,t}(w^n_{1,t}(i),w^n_{2,t}(i))]^{1-\eta} \tag{2.52}$$

$$\text{s.t.}\, w^n_{\tau,t+j}(i)=\begin{cases} w^n_{\tau,t+j-1}(i) & \text{with probability} \quad \lambda_\tau \\ w^{n*}_{\tau,t+j} & \text{with probability} \ \ 1-\lambda_\tau \end{cases}. \tag{2.53}$$

Here $w^{n*}_{\tau,t}$ is the optimized wage. The first-order condition can be written as

$$\eta\epsilon_{\tau,t}(w^{n*}_{1,t},w^n_{2,t}(i))J_{\tau,t}(w^{n*}_{1,t},w^n_{2,t}(i))=(1-\eta)\mu_{\tau,t}(w^{n*}_{1,t},w^n_{2,t}(i))H_{\tau,t}(w^{n*}_{1,t},w^n_{2,t}(i)), \tag{2.54}$$

where

$$\epsilon_{\tau,t}(w^n_{1,t}(i),w^n_{2,t}(i))\equiv p_t\frac{\partial H_{\tau,t}(w^n_{1,t}(i),w^n_{2,t}(i))}{\partial w^n_{\tau,t}(i)}, \tag{2.55}$$

$$\mu_{\tau,t}\left(w^n_{1,t}(i),w^n_{2,t}(i)\right)\equiv -p_t\frac{\partial J_{\tau,t}(w^n_{1,t}(i),w^n_{2,t}(i))}{\partial w^n_{\tau,t}(i)}. \tag{2.56}$$

We can rewrite the FOC as follows:

$$\chi_{\tau,t}\left(w^{n*}_{\tau,t},w^n_{\tau',t}\right)J_{\tau,t}\left(w^{n*}_{\tau,t},w^n_{\tau',t}\right)=[1-\chi_{\tau,t}(w^{n*}_{\tau,t},w^n_{\tau',t})]H_{\tau,t}\left(w^{n*}_{\tau,t},w^n_{\tau',t}\right), \tag{2.57}$$

where

$$\chi_{\tau,t}\left(w^{n*}_{\tau,t},w^n_{\tau',t}\right)\equiv\eta\left[\eta+(1-\eta)\frac{\mu_{\tau,t}\left(w^{n*}_{\tau,t},w^n_{\tau',t}\right)}{\epsilon_{\tau,t}}\right]^{-1}. \tag{2.58}$$

The effect of a rise in the real wage on the worker's surplus $\epsilon_{\tau,t}$, and minus the effect of a rise in the real wage on the firm's surplus $\mu_{\tau,t}\left(w^n_{1,t}(i),w^{n*}_{2,t}\right)$ are derived as follows:

$$\epsilon_{\tau,t} = 1 + \beta E_t \Lambda_{t,t+1} \lambda_\tau \frac{p_t}{p_{t+1}} \rho \epsilon_{\tau,t+1}, \tag{2.59}$$

$$\mu_{\tau,t}\left(w_{1,t}^n(i), w_{2,t}^{n*}\right) = 1 + \beta E_t \Lambda_{t,t+1} \lambda_\tau [\rho + x_{\tau,t+1}\left(w_{1,t+1}^n(i), w_{2,t}^{n*}\right)] \frac{p_t}{p_{t+1}} \mu_{\tau,t+1}\left(w_{1,t+1}^n(i), w_{2,t}^{n*}\right). \tag{2.60}$$

The assumptions of the probability of wage negotiations and the law of large numbers imply the evolution of the average nominal wage as follows:

$$w_{\tau,t+1}^n = (1-\lambda_\tau) w_{\tau,t+1}^n + \lambda_\tau \int_0^1 w_{\tau,t}^n(i) \frac{\rho + x_{\tau,t+1}(w_{\tau,t}^n(i))}{\rho + x_{\tau,t+1}} \frac{n_{\tau,t}(i)}{n_{\tau,t}} di. \tag{2.61}$$

2.6. Retailers

There is a continuum of monopolistically competitive retailers indexed by j on the unit interval. After retailers buy intermediate goods from the wholesale firms, they differentiate them with a technology that transforms one unit of intermediate goods into one unit of retail goods in order to resell them to the households. In addition, they set prices following Calvo (1983)'s manner.

Final goods, denoted with y_t, are a composite of individual retail goods, defined by the Dixit-Stiglitz function

$$y_t = \left[\int_0^1 y_t(j)^{\frac{\epsilon^p - 1}{\epsilon^p}} dj\right]^{\frac{\epsilon^p}{\epsilon^p - 1}}. \tag{2.62}$$

Cost minimization is conducive to the demand curve

$$y_t(j) = \left[\frac{p_t(j)}{p_t}\right]^{-\epsilon^p} y_t. \tag{2.63}$$

Here we define the consumer price index p_t as

$$p_t \equiv \left[\int_0^1 p_t(j)^{1-\epsilon^p} dj\right]^{\frac{1}{1-\epsilon^p}}. \tag{2.64}$$

Let $1 - \lambda_p$ be the probability that a firm adjusts its price. Firms not adjusting their target price simply posit the last-period price:

$$p_t(j) = p_{t-1}(j). \tag{2.65}$$

Reoptimizing retailers choose a target price, p_t^*, to maximize the following discounted stream of future profits:

$$E_t \sum_{s=0}^{\infty} \left(\lambda_p \beta\right)^s \Lambda_{t,t*s} \left[\frac{p_t^*}{p_{t+s}} - p_{t+s}^w\right] y_{t+s}(j). \tag{2.66}$$

The first order condition indicates the following New Keynesian Phillips Curve:

$$\pi_t = \varphi \hat{p}_t^w + \beta E_t \pi_{t+1}, \tag{2.67}$$

where

$$\varphi \equiv \frac{(1-\lambda_p)(1-\lambda_p\beta)}{\lambda_p}. \tag{2.68}$$

Since we normalize the relative price of final output at unity, the retailer's markup is given by

$$\mu_t^p = \frac{1}{p_t^w}. \tag{2.69}$$

Based on the optimization condition of the retailers, we obtain the markup in the steady state as

$$\mu_t^p = \frac{1}{p_t^w} = \frac{\epsilon^p}{\epsilon^p - 1}. \tag{2.70}$$

2.7. Wage and Hiring Dynamics

The evolution of the average wage has the form[6],

$$\widehat{w}_{\tau,t} = \gamma_\tau \widehat{w}_{\tau',t} + \gamma_b^\tau\left(\widehat{w}_{\tau,t-1} - \hat{\pi}_t\right) + \gamma_o^\tau \widehat{w}_{\tau,t}^o + \gamma_{\tau,f}^\tau\left(E_t\widehat{w}_{\tau,t+1} + E_t\hat{\pi}_{t+1}\right) + \gamma_{\tau',f}^\tau\left(E_t\widehat{w}_{\tau',t+1} + E_t\hat{\pi}_{t+1}\right), \tag{2.71}$$

[6] The details of derivations for equations in this subsection are shown in Appendix B.

where the economy target wage $\hat{w}^{o}_{\tau,t}$ satisfies

$$\hat{w}^{o}_{\tau,t} = \phi^{\tau}_{a}\left(\hat{p}^{w}_{t} + \hat{a}_{\tau,t}\right) + \left(\phi^{\tau}_{x} + \phi^{\tau}_{\tau,s}\right)E_t\hat{x}_{\tau,t+1} + \phi^{\tau}_{\tau',s}E_t\hat{x}_{\tau',t+1} + \phi^{\tau}_{b}\hat{b}_t + \phi^{\tau}_{\tau,s}E_t\hat{s}_{\tau,t+1} + \phi^{\tau}_{\tau',s}E_t\hat{s}_{\tau',t+1} + \left[\phi^{\tau}_{\tau,s} + \phi^{\tau}_{\tau',s} + \frac{\phi^{\tau}_{x}}{2}\right]E_t\hat{\Lambda}_{t,t+1} + \phi^{\tau}_{\chi}\hat{\chi}_{\tau,t} + \left[\phi^{\tau}_{\tau,s}(1-\chi_{\tau})^{-1} - \phi^{\tau}_{\chi}\rho\beta\right]E_t\hat{\chi}_{\tau,t+1} + \phi^{\tau}_{\tau',s}(1-\chi_{\tau})^{-1}E_t\hat{\chi}_{\tau',t+1}. \quad (2.72)$$

Then, the hiring dynamics are derived as

$$\hat{x}_{\tau,t} = X^{\tau}_{a}\left(\hat{p}^{w}_{t} + \hat{a}_{\tau,t}\right) - X^{\tau}_{w}\hat{w}_{\tau,t} + X^{\tau}_{\lambda}E_t\hat{\Lambda}_{t,t+1} + \beta E_t\hat{x}_{\tau,t+1}. \quad (2.73)$$

2.8. Resource Constraint

The final output is distributed for consumption, investment, government spending, hiring cost, and capital utilization cost:

$$y_t = c_t + i_t + g_t + \frac{\kappa}{2}\int_0^1 [x_{1,t}(i)]^2 n_{1,t-1}(i)di + \frac{\kappa}{2}\int_0^1 [x_{2,t}(i)]^2 n_{2,t-1}(i)di + A(v_t)k^{p}_{t-1}. \quad (2.74)$$

2.9. Government Spending and Monetary Policy

Government spending is simply proportional to output and obeys:

$$g_t = \epsilon^{g} y_t. \quad (2.75)$$

Here ϵ^{g} is assumed to be constant.

We compare welfare under alternative monetary policy rules. As usual, our baseline is the following Taylor rule with the interest-rate inertia,

$$\frac{r_{t+1}}{r} = \left(\frac{r_t}{r}\right)^{\rho^s}\left[\left(\frac{\pi_t}{\pi}\right)^{r_\pi}\left(\frac{y_t}{y}\right)^{r_y}\right]^{1-\rho^s}. \quad (2.76)$$

Here, ρ^s, r_π, and r_y are exogenous policy parameters. Based on implications of existing studies, including Galí (2012), that stabilizing wage inflations can improve welfare when we suppose labor disutility, the alternative rules include wage inflations as one of target variables

$$\frac{r_{t+1}}{r} = \left(\frac{r_t}{r}\right)^{\rho^s}\left[\left(\left[\frac{\pi_t}{\pi}\right]^{0.5}\left[\frac{\pi_t^w}{\pi^w}\right]^{0.5}\right)^{r_\pi}\left(\frac{y_t}{y}\right)^{r_y}\right]^{1-\rho^s}, \tag{2.77}$$

$$\frac{r_{t+1}}{r} = \left(\frac{r_t}{r}\right)^{\rho^s}\left[\left(\left[\frac{\pi_t}{\pi}\right]^{0.5}\left[\frac{\pi_{1,t}^w}{\pi_1^w}\right]^{0.5}\right)^{r_\pi}\left(\frac{y_t}{y}\right)^{r_y}\right]^{1-\rho^s}, \tag{2.78}$$

$$\frac{r_{t+1}}{r} = \left(\frac{r_t}{r}\right)^{\rho^s}\left[\left(\left[\frac{\pi_t}{\pi}\right]^{0.5}\left[\frac{\pi_{2,t}^w}{\pi_2^w}\right]^{0.5}\right)^{r_\pi}\left(\frac{y_t}{y}\right)^{r_y}\right]^{1-\rho^s}. \tag{2.79}$$

2.10. Welfare Measure

To obtain the welfare implications of alternative monetary policy rules, we define the welfare measure following Kollmann (2004). A second-order expansion of the household's utility function $W(\cdot)$ around the steady state yields

$$EW(c_t, n_{1,t}, n_{2,t}) \cong ln\, c - \Omega\frac{n_1^{1+\iota}}{1+\iota} - \Omega\frac{n_2^{1+\iota}}{1+\iota} - \widetilde{\Psi}, \tag{2.80}$$

where

$$\widetilde{\Psi} \equiv E[\hat{c}_t] - \Omega n_1^{1+\iota}E[\hat{n}_{1,t}] - \Omega n_2^{1+\iota}E[\hat{n}_{2,t}] + \frac{1}{2}V[\hat{c}_t] + \frac{\Omega\iota}{2}\{n_1^{1+\iota}V[\hat{n}_{1,t}] + n_2^{1+\iota}V[\hat{n}_{2,t}]\}, \tag{2.81}$$

such that E and V indicate the expectation and variance operators, respectively. Since we use the standard first-order log-linearization method and assume zero means for all disturbances, only the second moments are effective on the welfare.

Now we define the welfare loss caused by macroeconomic volatility as the permanent relative change in consumption compared to the steady state. Accordingly, the welfare loss has the form,

$$\Psi = 1 - c^{-\widetilde{\Psi}}, \tag{2.82}$$

where

$$W([1-\Psi]c, n_1, n_2) = EW(c_t, n_{1,t}, n_{2,t}). \tag{2.83}$$

3. Impulse responses and second moments

3.1. Parameterization

We compare the welfare effects under alternative monetary policy rules considering the heterogeneity in wage rigidity. Before getting into the heterogeneous case, we will analyze the result for the homogeneous case. For this, we set $\lambda_1 = \lambda_2 = 0.75$, indicating that firms and workers have the chance to renegotiate wages once a year on average. We set $\eta = 0.5$ following Gertler and Trigari (2009). Further, we follow them for the calibration of total job finding rate and set $s_1 + s_2 = 0.95$. For the case of heterogeneous wage rigidity, we set $\lambda_1 = 0.25$ and $\lambda_2 = 0.75$, indicating that type-1 workers' wages are more flexible than type-2 workers.

The other parameter values presented in Table 1 are common for all analyses, for which we follow the convention. The objective discount factor β is set equal to 0.99 to imply that the annual return on bonds is around 4% in the steady state. We set $\epsilon^p = 6$ so that the steady-state markup on price is equal to 1.2. For ρ^z, ρ, ϵ^g, η_k, and ψ_v, we follow Gertler et al. (2008). We follow their estimation also for κ and b, and set $\kappa = 2.81$ and $b = 0.723$, respectively. We set the degree of price rigidity λ_p as $\lambda_p = 0.75$ to indicate that firms have the chance to adjust their prices once a year on average. The monetary policy parameters are supposed to be $\rho^s = 0.773$, $r_\pi = 2.006$, and $\phi_y = 0.332$. The steady state values of the utilization rate v and the cost of capital accumulation $A(v)$ are set equal to unity and zero, respectively.

3.2. Impulse responses

Figure 1 presents the impulse responses of selected variables to the 1% positive shock on the labor productivity in the homogeneous case[7]. Positive labor productivity shocks are conducive to decreases in employment variables as firms cut workers. Accordingly, unemployment rises; output rises and CPI inflation falls because firms become more efficient in production. As a response to the decline in CPI inflation, nominal interest rate falls regardless of monetary policy rules. Nominal wage inflations decline as a result of declines in the labor demands of firms. Through the Euler equation, consumption rises.

As we do not suppose heterogeneity, there are no differences between dynamics of variables among rules (2.77), (2.78), and (2.79). Here we find that wage inflations converge to their steady state values faster, and their volatilities become smaller with these three rules than the standard Taylor rule (2.76). This is straightforward because the former rules split some weight for wage inflations from CPI inflation.

[7] All the log-linearized equations are presented in Appendix C.

Employment variables are also stabilized more by rules with wage inflation targets. This is because the stabilization of wages can stabilize employment by stabilizing the labor demands of firms. Capital input becomes more volatile in the cases with (2.77), (2.78), and (2.79) than with (2.76). This is because capital input should be adjusted instead so that labor input remains stable given the demand on goods that firms produce. Consumption is stabilized more by the standard Taylor rule as it focuses more on CPI inflation, to which the household refers when it decides on consumption.

Figure 2 presents the responses in the case with the heterogeneity in wage rigidity. Directions of responses are almost the same as in Figure 1, indicating that the heterogeneity in wage rigidity does not affect macroeconomic dynamics qualitatively. However, we find some significant differences between them in quantitative senses. Here, we should remember that type-1 wages are more flexible than type-2 wages. Trend dynamics are not different among the four rules.

Now let us interpret the difference between (2.77), (2.78), and (2.79). Here, we find that policy rule (2.78), namely the rule with more flexible wage target, stabilizes both types of wage inflations and employment variables more than the other rules. Why can this rule stabilize both types of wages and employment variables although it focuses just on type-1 wages? This is because both types of wages are correlated positively as a result of bargaining behaviors. If a given type of worker changes wages much more than another type of worker, the former employment becomes more volatile than the latter. To avoid this situation, each type of worker changes wages with comparable magnitude to other types. As a result, we find that significant positive correlation between two types of wage inflations, and find that a central bank can stabilize both types of wage inflations by targeting either one. Further, we should note that type-1 wage inflation is more volatile as it is more flexible. As a result, stabilizing more rigid wage inflation, namely Rule (2.79), becomes not to enough to stabilize more flexible and volatile ones; in this case, Rule (2.78) is found to be better to stabilize wage inflations and employment variables.

3.3. Second moments and welfare implications

Table 2 shows standard deviations and welfare losses for each monetary policy rule in the homogeneous and heterogeneous cases. In this table, standard deviations and welfare losses are normalized by levels with the rule (2.76). We also consider alternative parameterizations for Ω and ι. The first parameterization, $\Omega = 0$, indicates the zero-weight on labor disutility in the household's utility, as in Gertler et al. (2008). Following Galí (2011) and Gertler and Karadi (2011), respectively, we also try to obtain welfare implications with parameterizations $(\Omega, \iota) = (1,5)$ and $(\Omega, \iota) = (3.409, 0.276)$.

Welfare implications are fully consistent with the impulse responses. In the homogeneous case, we find that the standard deviation of the consumption is increased by introducing wage inflations into the policy rule. In contrast, standard deviations of employment and wage inflation variables decrease. In cases with parameterizations $\Omega = 0$ and $(\Omega, \iota) = (1, 5)$, the standard Taylor rule (2.76) is found to be the best among the four policy rules. In contrast, when we posit heavier weight on employment such as $(\Omega, \iota) = (3.409, 0.276)$, policy rules with wage inflations become better than the standard ones without wage inflations.

Heterogeneity in wage rigidity is conducive to the variety of results among cases with alternative policy rules. The trend of changes in standard deviations is almost the same as the homogeneous case, even if we introduce the heterogeneity. The policy rule with type-1 wage inflation, namely the inflation of more flexible wage, is always found to be the best among rules (2.77), (2.78), and (2.79). However, the standard Taylor rule without wage inflations dominates these three rules except for the case with parameterization of $(\Omega, \iota) = (3.409, 0.276)$. In this case, the policy rule (2.78) becomes the best among the four.

Our results have important implications for monetary policy implementations. It is clear that there is heterogeneity in wage rigidity as suggested by the empirical studies, including ones we have reviewed in the introduction. Further, our simulation results imply that the welfare results can differ depending on rules for monetary policy implementations. This means that there is room for discussing heterogeneity in wage rigidity for better policy consequences. As our parameterization applies mostly to exogenous parameters, the results should not be applied for all countries facing the heterogeneity in wage rigidity. However, we have found it is clear that this type of heterogeneity affects the desirability of monetary policy arrangements.

4. Concluding remarks

In this paper, we have constructed a DSGE model with heterogeneity in wage rigidity, and compared alternative monetary policy rules on the basis of welfare. For the labor productivity shock, monetary policy rules with considerations of wage inflations improve the welfare by stabilizing employment. They are found to be better than the standard policy rule without wage inflations when the weight on labor disutility in the household's utility is sufficiently large. Further, we have found that a policy rule with consideration for more flexible wage inflation could be better than a rule with less-flexible wage inflation target.

It is natural that the results raise questions about the optimal monetary policy. Can we find optimal weights on wage inflations analytically? How can we suggest a simple policy rule which replicates the optimal allocation? These questions are important for future research.

Acknowledgements

We have benefited from discussions with and the comments of Fumio Dei, Yuichi Furukawa, Hugh Patrick, and David Weinstein. We also would like to thank the participants at 10th Biennial Pacific Rim Conference of Western Economic Association International (WEAI) in March 2013 and SIBR-Thammasat Bangkok Conference on Interdisciplinary Business and Economics Research in June 2013, and the seminar participants at Center on Japanese Economy and Business (CJEB) at Columbia Business School and Bangkok Research Center, IDE-JETRO. All remaining errors are ours. This work was supported by Nanzan University Pache Research Subsidy I-A-2 for the 2013 academic year.

Appendix A. Hiring and Dismissal in the Steady State

In the steady state, we have relations

$$\frac{n_\tau}{u} = \frac{s_\tau}{x_\tau} \tag{A.1}$$

and

$$s_\tau = \frac{m_\tau}{u}. \tag{A.2}$$

Combining them, we obtain

$$n_\tau = \frac{m_\tau}{x_\tau} \tag{A.3}$$

which yields the following relations

$$\rho + x_\tau = 1. \tag{A.4}$$

This gives

$$n_\tau(1-\rho) = m_\tau. \tag{A.5}$$

This relation implies that the number of new hires is equal to the number of workers fired in the steady state. Accordingly, numbers and shares of each type of workers are constants in the steady state.

Appendix B. Derivation of Wage and Hiring Dynamics

Appendix B.1. Wage Dynamics

The first order condition of Nash Bargaining (2.57) can be log-linearized as

$$\hat{J}_{\tau,t}\left(w_{\tau,t}^{n*}, w_{\tau',t}^{n}(i)\right) + \left(1-\chi_\tau\left(w_\tau^{n*}, w_{\tau'}^{n}\right)\right)^{-1}\hat{\chi}_{\tau,t}\left(w_{\tau,t}^{n*}, w_{\tau',t}^{n}(i)\right) = \widehat{H}_{\tau,t}\left(w_{\tau,t}^{n*}, w_{\tau',t}^{n}(i)\right). \tag{B.1}$$

Note that $\tau = \{1,2\}$, $\tau' = \{1,2\}$, and $\tau \neq \tau'$. Combining equations we obtain

$$\frac{p^w a_\tau}{J_\tau}\left(\hat{p}_t^w + \hat{a}_{\tau,t}(i)\right) - \frac{w_\tau}{J_\tau}\{\widehat{w}_{\tau,t}^* + \beta\lambda_\tau\mu_\tau(\widehat{w}_{\tau,t}^* - E_t\hat{\pi}_{t+1} - E_t\widehat{w}_{\tau,t+1}^*)\} +$$

$$x_\tau\beta\left\{E_t\hat{x}_{\tau,t+1}\left(w_{\tau,t+1}^{n*}, w_{\tau',t+1}^{n}(i)\right) + \frac{1}{2}E_t\hat{\Lambda}_{t,t+1}\right\} + (1-\chi_\tau)^{-1}\hat{\chi}_{\tau,t}\left(w_{\tau,t}^{n*}, w_{\tau',t}^{n}(i)\right) = \frac{w_\tau}{H_\tau}\{\widehat{w}_{\tau,t}^* +$$

$$\beta\rho_\tau\lambda_\tau\epsilon_\tau(\widehat{w}_{\tau,t}^* - E_t\hat{\pi}_{t+1} - E_t\widehat{w}_{\tau,t+1}^*)\} - \frac{b}{H_\tau}\hat{b}_t - \beta s_\tau\{\hat{s}_{\tau,t+1} + \widehat{H}_{x,t+1}^{\tau} + E_t\hat{\Lambda}_{t,t+1}\} -$$

$$\beta s_{\tau'}\frac{H_{\tau'}}{H_\tau}\{\hat{s}_{\tau',t+1} + \widehat{H}_{x,t+1}^{\tau'} + E_t\hat{\Lambda}_{t,t+1}\} + \beta\rho_\tau(1-\chi_\tau)^{-1}E_t\hat{x}_{\tau,t+1}\left(w_{\tau,t+1}^{n*}, w_{\tau',t+1}^{n}(i)\right). \tag{B.2}$$

We can rewrite this as follows:

$$\widehat{w}_{\tau,t}^* + \psi_\tau(\widehat{w}_{\tau,t}^* - E_t\hat{\pi}_{t+1} - E_t\widehat{w}_{\tau,t+1}^*) =$$

$$\chi_\tau p^w\frac{a_\tau}{w_\tau}\left(\hat{p}_t^w + \hat{a}_{\tau,t}(i)\right) + \chi_\tau x_\tau\beta\frac{J_\tau}{w_\tau}\left\{E_t\hat{x}_{\tau,t+1}\left(w_{\tau,t+1}^{n*}, w_{\tau',t+1}^{n}(i)\right) + \frac{1}{2}E_t\hat{\Lambda}_{t,t+1}\right\} + (1-$$

$$\chi_\tau)\frac{b}{w_\tau}\hat{b}_t + (1-\chi_\tau)\frac{H_\tau}{w_\tau}\beta s_\tau\{\hat{s}_{\tau,t+1} + \widehat{H}_{x,t+1}^{\tau} + E_t\hat{\Lambda}_{t,t+1}\} + (1-\chi_\tau)\frac{H_{\tau'}}{w_\tau}\beta s_{\tau'}\{\hat{s}_{\tau',t+1} + \widehat{H}_{x,t+1}^{\tau'} +$$

$$E_t\hat{\Lambda}_{t,t+1}\} + \chi_\tau \frac{J_\tau}{w_\tau}(1-\chi_\tau)^{-1}\left[\hat{\chi}_{\tau,t}\left(w^{n*}_{\tau,t}, w^{n}_{\tau',t}(i)\right) - \beta\rho_\tau\hat{\chi}_{\tau,t+1}\left(w^{n*}_{\tau,t+1}, w^{n}_{\tau',t+1}(i)\right)\right]. \quad \text{(B.3)}$$

Here we used

$$\frac{J_\tau}{H_\tau} = \frac{1-\chi_\tau}{\chi_\tau}, \quad \text{(B.4)}$$

$$\psi_\tau = \chi_\tau\beta\lambda_\tau\mu_\tau + (1-\chi_\tau)\rho_\tau\beta\lambda_\tau\epsilon_\tau. \quad \text{(B.5)}$$

Further,

$$\widehat{w}^*_{\tau,t} = (1-\zeta_\tau)w^o_{\tau,t}\left(w^{n*}_{\tau,t}, w^{n}_{\tau',t}\right) + \zeta_\tau E_t\widehat{\pi}_{t+1} + \zeta_\tau E_t\widehat{w}^*_{\tau,t+1}, \quad \text{(B.6)}$$

where

$$\zeta_\tau \equiv \frac{\psi_\tau}{1+\psi_\tau}, \quad \text{(B.7)}$$

$$w^o_{\tau,t}\left(w^{n*}_{\tau,t}, w^{n}_{\tau',t}\right) \equiv \phi^\tau_a\left(\hat{p}^w_t + \hat{a}_{\tau,t}(i)\right) + \phi^\tau_x\left\{E_t\hat{x}_{\tau,t+1}\left(w^{n*}_{\tau,t+1}, w^{n}_{\tau',t+1}(i)\right) + \frac{1}{2}E_t\widehat{\Lambda}_{t,t+1}\right\} +$$
$$\phi^\tau_b\hat{b}_t + \phi^\tau_{\tau,s}\left\{\hat{s}_{\tau,t+1} + \widehat{H}^\tau_{x,t+1} + E_t\widehat{\Lambda}_{t,t+1}\right\} + \phi^\tau_{\tau',s}\left\{\hat{s}_{\tau',t+1} + \widehat{H}^{\tau'}_{x,t+1} + E_t\widehat{\Lambda}_{t,t+1}\right\} +$$
$$\phi^\tau_\chi\left[\hat{\chi}_{\tau,t}\left(w^{n*}_{\tau,t}, w^{n}_{\tau',t}(i)\right) - \beta\rho_\tau\xi_\tau\hat{\chi}_{\tau,t+1}\left(w^{n*}_{\tau,t+1}, w^{n}_{\tau',t+1}(i)\right)\right], \quad \text{(B.8)}$$

and

$$\phi^\tau_a \equiv \chi_\tau p^w\frac{a_\tau}{w_\tau}, \quad \text{(B.9)}$$

$$\phi^\tau_x \equiv \chi_\tau x_\tau\beta\frac{J_\tau}{w_\tau}, \quad \text{(B.10)}$$

$$\phi^\tau_b \equiv (1-\chi_\tau)\frac{b}{w_\tau}, \quad \text{(B.11)}$$

$$\phi^\tau_{\tau,s} \equiv (1-\chi_\tau)\frac{H_\tau}{w_\tau}\beta s_\tau, \quad \text{(B.12)}$$

$$\phi^\tau_{\tau',s} \equiv (1-\chi_\tau)\frac{H_{\tau'}}{w_\tau}\beta s_{\tau'}, \quad \text{(B.13)}$$

$$\phi^\tau_\chi \equiv \chi_\tau\frac{J_\tau}{w_\tau}(1-\chi_\tau)^{-1}. \quad \text{(B.14)}$$

Appendix B.2. The Spillover Effects

Log-linearizing $\hat{x}_{\tau,t+1}(w_{\tau,t}^{n*}) - \hat{x}_{\tau,t+1}(w_{\tau,t}^{n})$ we obtain

$$E_t\left[\hat{x}_{\tau,t+1}\left(w_{\tau,t}^{n*}, w_{\tau',t+1}^{n}(i)\right) - \hat{x}_{\tau,t+1}\left(w_{\tau,t}^{n}, w_{\tau',t+1}^{n}(i)\right)\right] = -N_\tau \mu_\tau E_t(\widehat{w}_{\tau,t}^{*} - \widehat{w}_{\tau,t}). \quad \text{(B.15)}$$

Note that

$$\hat{\mu}_{\tau,t}(w_{\tau,t}^{n*}, w_{\tau',t}^{n}) = \beta\lambda_\tau x_\tau \hat{x}_{\tau,t+1}(w_{\tau,t}^{n*}, w_{\tau',t+1}^{n}) + \beta\lambda_\tau[\hat{\mu}_{\tau,t+1}(w_{\tau,t}^{n*}, w_{\tau',t+1}^{n}) + E_t\widehat{\Lambda}_{t,t+1} - E_t\widehat{\pi}_{t,t+1}], \quad \text{(B.16)}$$

$$\hat{\mu}_{\tau,t}(w_{\tau,t}^{n}, w_{\tau',t}^{n}) = \beta\lambda_\tau x_\tau \hat{x}_{\tau,t+1}(w_{\tau,t}^{n}, w_{\tau',t+1}^{n}) + \beta\lambda_\tau[\hat{\mu}_{\tau,t+1}(w_{\tau,t}^{n}, w_{\tau',t+1}^{n}) + E_t\widehat{\Lambda}_{t,t+1} - E_t\widehat{\pi}_{t,t+1}]. \quad \text{(B.17)}$$

By differentiating both sides, we obtain the following relation

$$\hat{\mu}_{\tau,t}(w_{\tau,t}^{n*}, w_{\tau',t}^{n}) - \hat{\mu}_{\tau,t}(w_{\tau,t}^{n}, w_{\tau',t}^{n}) = \beta\lambda_\tau x_\tau[\hat{x}_{\tau,t+1}(w_{\tau,t}^{n*}, w_{\tau',t+1}^{n}) - \hat{x}_{\tau,t+1}(w_{\tau,t}^{n}, w_{\tau',t+1}^{n})] + \beta\lambda_\tau[\hat{\mu}_{\tau,t+1}(w_{\tau,t}^{n*}, w_{\tau',t+1}^{n}) - \hat{\mu}_{\tau,t+1}(w_{\tau,t}^{n}, w_{\tau',t+1}^{n})] = -\beta\lambda_\tau x_\tau N_\tau \mu_\tau E_t[\widehat{w}_{\tau,t}^{*} - \widehat{w}_{\tau,t}] + \beta\lambda_\tau[\hat{\mu}_{\tau,t+1}(w_{\tau,t}^{n*}, w_{\tau',t+1}^{n}) - \hat{\mu}_{\tau,t+1}(w_{\tau,t}^{n}, w_{\tau',t+1}^{n})]. \quad \text{(B.18)}$$

Substituting forward we obtain

$$\hat{\mu}_{\tau,t}(w_{\tau,t}^{n*}, w_{\tau',t}^{n}) - \hat{\mu}_{\tau,t}(w_{\tau,t}^{n}, w_{\tau',t}^{n}) = -\beta\lambda_\tau x_\tau N_\tau \mu_\tau[1 + \beta\lambda_\tau + (\beta\lambda_\tau)^2 + \cdots]E_t[\widehat{w}_{\tau,t}^{*} - \widehat{w}_{\tau,t}] = -\beta\lambda_\tau x_\tau N_\tau \mu_\tau^2 E_t[\widehat{w}_{\tau,t}^{*} - \widehat{w}_{\tau,t}]. \quad \text{(B.19)}$$

Based on the definition of $\chi_{\tau,t}$, the following log-linearized relations can be obtained

$$\hat{\chi}_{\tau,t}(w_{\tau,t}^{n*}, w_{\tau',t}^{n}) = (1 - \chi_\tau)[\hat{\epsilon}_{\tau,t} - \hat{\mu}_{\tau,t}(w_{\tau,t}^{n*}, w_{\tau',t}^{n})], \quad \text{(B.20)}$$

$$\hat{\chi}_{\tau,t}(w_{\tau,t}^{n}, w_{\tau',t}^{n}) = (1 - \chi_\tau)[\hat{\epsilon}_{\tau,t} - \hat{\mu}_{\tau,t}(w_{\tau,t}^{n}, w_{\tau',t}^{n})]. \quad \text{(B.21)}$$

By differentiating both sides

$$\hat{\chi}_{\tau,t}(w_{\tau,t}^{n*}, w_{\tau',t}^{n}) - \hat{\chi}_{\tau,t}(w_{\tau,t}^{n}, w_{\tau',t}^{n}) = -(1 - \chi_\tau)[\hat{\mu}_{\tau,t}(w_{\tau,t}^{n*}, w_{\tau',t}^{n}) - \hat{\mu}_{\tau,t}(w_{\tau,t}^{n}, w_{\tau',t}^{n})] = (1 - \chi_\tau)\beta\lambda_\tau x_\tau N_\tau \mu_\tau^2 E_t(\widehat{w}_{\tau,t}^{*} - \widehat{w}_{\tau,t}). \quad \text{(B.22)}$$

Log-linearization of $E_t[H_{\tau,t+1}(w_{\tau,t+1}^{n*}, w_{\tau',t+1}^{n}) - H_{\tau,t+1}(w_{\tau,t+1}^{n}, w_{\tau',t+1}^{n})]$ implies

$$H_\tau \widehat{H}_{\tau,t+1}\left(w^{n*}_{\tau,t+1}, w^{n}_{\tau',t+1}\right) - H_\tau \widehat{H}_{\tau,t+1}\left(w^{n*}_{\tau,t+1}, w^{n}_{\tau',t+1}\right) = w^n_\tau H'_\tau(\cdot) E_t[\widehat{w}^{n*}_{\tau,t+1} - \widehat{w}^{n}_{\tau,t+1}]. \quad (B.23)$$

By using the steady-state relations, this can be rewritten as

$$\widehat{H}_{\tau,t+1}\left(w^{n*}_{\tau,t+1}, w^{n}_{\tau',t+1}\right) - \widehat{H}_{\tau,t+1}\left(w^{n}_{\tau,t+1}, w^{n}_{\tau',t+1}\right) = \frac{1-\chi_\tau}{\chi_\tau} N_\tau \epsilon_\tau E_t[\widehat{w}^{n*}_{\tau,t+1} - \widehat{w}^{n}_{\tau,t+1}]. \quad (B.24)$$

Taking log-linearization, we have $E_t[J_{\tau,t+1}\left(w^{n*}_{\tau,t+1}, w^{n}_{\tau',t+1}\right) - J_{\tau,t+1}\left(w^{n}_{\tau,t+1}, w^{n}_{\tau',t+1}\right)]$

$$J_\tau \hat{J}_{\tau,t+1}\left(w^{n*}_{\tau,t+1}, w^{n}_{\tau',t+1}\right) - J_\tau \hat{J}_{\tau,t+1}\left(w^{n}_{\tau,t+1}, w^{n}_{\tau',t+1}\right) = w^n_\tau J'_\tau(\cdot) E_t\left[\widehat{w}^{n*}_{\tau,t+1} - \widehat{w}^{n}_{\tau,t+1}\right], \quad (B.25)$$

which is rewritten with the steady-state relations as

$$\hat{J}_{\tau,t+1}\left(w^{n*}_{\tau,t+1}, w^{n}_{\tau',t+1}\right) - \hat{J}_{\tau,t+1}\left(w^{n}_{\tau,t+1}, w^{n}_{\tau',t+1}\right) = -N_\tau \mu_\tau E_t\left[\widehat{w}^{n*}_{\tau,t+1} - \widehat{w}^{n}_{\tau,t+1}\right]. \quad (B.26)$$

The first order condition of Nash Bargaining implies

$$\chi_{\tau,t+1}\left(w^{n*}_{\tau,t+1}, w^{n}_{\tau',t+1}\right) J_{\tau,t+1}\left(w^{n*}_{\tau,t+1}, w^{n}_{\tau',t+1}\right) = [1 - \chi_{\tau,t+1}\left(w^{n*}_{\tau,t+1}, w^{n}_{\tau',t+1}\right)] H_{\tau,t+1}\left(w^{n*}_{\tau,t+1}, w^{n}_{\tau',t+1}\right). \quad (B.27)$$

Taking log-linearization, we obtain

$$E_t \hat{J}_{\tau,t+1}\left(w^{n*}_{\tau,t+1}, w^{n}_{\tau',t+1}\right) + \left(1 + \frac{H_\tau}{J_\tau}\right) E_t \hat{\chi}_{\tau,t+1}\left(w^{n*}_{\tau,t+1}, w^{n}_{\tau',t+1}\right) = E_t \widehat{H}_{\tau,t+1}\left(w^{n*}_{\tau,t+1}, w^{n}_{\tau',t+1}\right) \quad (B.28)$$

or

$$E_t \hat{J}_{\tau,t+1}\left(w^{n*}_{\tau,t+1}, w^{n}_{\tau',t+1}\right) + (1-\chi_\tau)^{-1} E_t \hat{\chi}_{\tau,t+1}\left(w^{n*}_{\tau,t+1}, w^{n}_{\tau',t+1}\right) = E_t \widehat{H}_{\tau,t+1}\left(w^{n*}_{\tau,t+1}, w^{n}_{\tau',t+1}\right). \quad (B.29)$$

Combining equations, we obtain

$$\hat{J}_{\tau,t+1}\left(w^{n}_{\tau,t+1}, w^{n}_{\tau',t+1}\right) + (1-\chi_\tau)^{-1} E_t \hat{\chi}_{\tau,t+1}\left(w^{n}_{\tau,t+1}, w^{n}_{\tau',t+1}\right) = E_t \widehat{H}_{\tau,t+1}\left(w^{n}_{\tau,t+1}, w^{n}_{\tau',t+1}\right) + \Gamma_\tau E_t\left[\widehat{w}^{n*}_{\tau,t+1} - \widehat{w}^{n}_{\tau,t+1}\right], \quad (B.30)$$

where

$$\Gamma_\tau \equiv [1 - \eta_\tau x_\tau \beta \lambda_\tau \mu_\tau]\eta_\tau^{-1}\mu_\tau N_\tau. \tag{B.31}$$

Finally, by using $\hat{J}_{\tau,t+1}(w^n_{\tau,t+1}, w^n_{\tau',t+1}) = \hat{x}_{\tau,t+1}(w^n_{\tau,t+1}, w^n_{\tau',t+1})$,

$$\begin{aligned}&\widehat{H}_{\tau,t+1}(w^n_{\tau,t+1}, w^n_{\tau',t+1}) = \\ &E_t\hat{x}_{\tau,t+1}(w^n_{\tau,t+1}, w^n_{\tau',t+1}) - \Gamma_\tau E_t[\widehat{w}^{n*}_{\tau,t+1} - \widehat{w}^n_{\tau,t+1}] + (1-\chi_\tau)^{-1}\hat{\chi}_{\tau,t+1}(w^n_{\tau,t+1}, w^n_{\tau',t+1}),\end{aligned} \tag{B.32}$$

or

$$\begin{aligned}&\widehat{H}_{\tau,t+1}(w^n_{\tau,t+1}, w^n_{\tau',t+1}) = \\ &E_t\hat{x}_{\tau,t+1}(w^n_{\tau,t+1}, w^n_{\tau',t+1}) - \Gamma_\tau E_t[\widehat{w}^{*}_{\tau,t+1} - \widehat{w}_{\tau,t+1}] + (1-\chi_\tau)^{-1}\hat{\chi}_{\tau,t+1}(w^n_{\tau,t+1}, w^n_{\tau',t+1}).\end{aligned} \tag{B.33}$$

Using relations obtained, we can rewrite $\widehat{w}^o_{\tau,t}(\cdot)$ as follows:

$$\begin{aligned}&\widehat{w}^o_{\tau,t}(w^{n*}_{\tau,t}, w^n_{\tau',t}) = \widehat{w}^o_{\tau,t} + \phi^\tau_x\{E_t\hat{x}_{\tau,t+1}(w^{n*}_{\tau,t+1}, w^n_{\tau',t+1}) - E_t\hat{x}_{\tau,t+1}(w^n_{\tau,t+1}, w^n_{\tau',t+1})\} + \\ &\phi^\tau_\chi[\hat{\chi}_{\tau,t}(w^{n*}_{\tau,t}, w^n_{\tau',t}) - \hat{\chi}_{\tau,t}(w^n_{\tau,t}, w^n_{\tau',t})] - \\ &\phi^\tau_\chi\rho_\tau\beta[\hat{\chi}_{\tau,t+1}(w^{n*}_{\tau,t+1}, w^n_{\tau',t+1}) - \hat{\chi}_{\tau,t+1}(w^n_{\tau,t+1}, w^n_{\tau',t+1})] - \phi^\tau_{\tau,s}\Gamma_\tau E_t[\widehat{w}^*_{\tau,t+1} - \widehat{w}_{\tau,t+1}] - \\ &\phi^\tau_{\tau',s}\Gamma_{\tau'}E_t[\widehat{w}^*_{\tau',t+1} - \widehat{w}_{\tau',t+1}]\end{aligned} \tag{B.34}$$

where

$$\begin{aligned}&\widehat{w}^o_{\tau,t} \equiv \\ &\phi^\tau_a(\hat{p}^w_t + \hat{a}_{\tau,t}) + (\phi^\tau_x + \phi^\tau_{\tau,s})E_t\hat{x}_{\tau,t+1}(w^n_{\tau,t+1}, w^n_{\tau',t+1}) + \phi^\tau_{\tau',s}E_t\hat{x}_{\tau',t+1}(w^n_{\tau,t+1}, w^n_{\tau',t+1}) + \\ &\phi^\tau_b\hat{b}_t + \phi^\tau_{\tau,s}\hat{s}_{\tau,t+1} + \phi^\tau_{\tau',s}\hat{s}_{\tau',t+1} + \left[\phi^\tau_{\tau,s} + \phi^\tau_{\tau',s} + \frac{\phi^\tau_x}{2}\right]E_t\hat{\Lambda}_{t,t+1} + \phi^\tau_\chi\hat{\chi}_{\tau,t}(w^n_{\tau,t}, w^n_{\tau',t}) + \\ &[\phi^\tau_{\tau,s}(1-\chi_\tau)^{-1} - \phi^\tau_\chi\rho_\tau\beta]\hat{\chi}_{\tau,t+1}(w^n_{\tau,t+1}, w^n_{\tau',t+1}) + \phi^\tau_{\tau',s}(1-\chi_{\tau'})^{-1}\hat{\chi}_{\tau',t+1}(w^n_{\tau,t+1}, w^n_{\tau',t+1}).\end{aligned} \tag{B.35}$$

Here we used the following relation

$$\widehat{H}^\tau_{x,t+1} = \widehat{H}_{\tau,t+1}(w^n_{\tau,t+1}, w^n_{\tau',t+1}). \tag{B.36}$$

Finally,

$$\widehat{w}^o_{\tau,t}(w^{n*}_{\tau,t}, w^n_{\tau',t}) =$$

$$\widehat{w}^{o}_{\tau,t} + \frac{\zeta_1^\tau}{1-\zeta_\tau} E_t\left[\widehat{w}_{\tau,t+1} - \widehat{w}^{*}_{\tau,t+1}\right] + \frac{\zeta_2^\tau}{1-\zeta_\tau} E_t\left[\widehat{w}_{\tau',t+1} - \widehat{w}^{*}_{\tau',t+1}\right] + \frac{\zeta_3^\tau}{1-\zeta_\tau} E_t\left[\widehat{w}_{\tau,t+1} - \widehat{w}^{*}_{\tau,t+1}\right] \quad \text{(B.37)}$$

where

$$\zeta_1^\tau \equiv \left(\phi_\chi^\tau \rho\beta(1-\chi_\tau)\beta\lambda_\tau x_\tau N_\tau \mu_\tau^2 + \phi_x^\tau \frac{\mu_\tau w_\tau}{J_\tau} + \phi_{\tau,s}^\tau \Gamma_\tau\right)(1-\zeta_\tau), \quad \text{(B.38)}$$

$$\zeta_2^\tau \equiv \phi_{\tau',s}^\tau \Gamma_{\tau'}(1-\zeta_\tau), \quad \text{(B.39)}$$

$$\zeta_3^\tau \equiv -[\phi_\chi^\tau(1-\chi_\tau)\beta\lambda_\tau x_\tau N_\tau \mu_\tau^2](1-\zeta_\tau). \quad \text{(B.40)}$$

Based on the definition of the wage index and the wage-rigidity assumption, we obtain the following result:

$$\widehat{w}_{\tau,t} = \gamma_\tau \widehat{w}_{\tau',t} + \gamma_b^\tau\left(\widehat{w}_{\tau,t-1} - \widehat{\pi}_t\right) + \gamma_o^\tau \widehat{w}^{o}_{\tau,t} + \gamma_{\tau,f}^\tau\left(E_t\widehat{w}_{\tau,t+1} + E_t\widehat{\pi}_{t+1}\right) + \gamma_{\tau',f}^\tau\left(E_t\widehat{w}_{\tau',t+1} + E_t\widehat{\pi}_{t+1}\right) \quad \text{(B.41)}$$

where

$$\phi_\tau \equiv \frac{1+\lambda_\tau(\zeta_\tau - \zeta_1^\tau + \zeta_3^\tau)}{1-\lambda_\tau}, \quad \text{(B.42)}$$

$$\gamma_\tau \equiv \frac{\zeta_2^\tau}{\phi_\tau}\frac{\lambda_{\tau'}}{1-\lambda_{\tau'}}, \quad \text{(B.43)}$$

$$\gamma_b^\tau \equiv \frac{1+\zeta_3^\tau}{\phi_\tau}\frac{\lambda_\tau}{1-\lambda_\tau}, \quad \text{(B.44)}$$

$$\gamma_o^\tau \equiv \frac{1-\zeta_\tau}{\phi_\tau}, \quad \text{(B.45)}$$

$$\gamma_{\tau,f}^\tau \equiv \frac{\zeta_\tau - \zeta_1^\tau\lambda_\tau}{(1-\lambda_\tau)\phi_\tau}, \quad \text{(B.46)}$$

$$\gamma_{\tau',f}^\tau \equiv -\frac{\zeta_2^\tau\lambda_{\tau'}}{(1-\lambda_{\tau'})\phi_\tau}. \quad \text{(B.47)}$$

Appendix B.3. Hiring Dynamics

By log-linearizing the first order conditions with respect to hiring rates, we obtain the following relation:

$$p^w a_\tau\left(\hat{p}_t^w + \hat{a}_{\tau,t}\right) = w_\tau \hat{w}_{\tau,t} + \kappa x_\tau \hat{x}_{\tau,t} - x_\tau \beta\kappa \hat{x}_{\tau,t-1} - \beta\kappa\left[\frac{1}{2}x_\tau^2 + \rho_\tau x_\tau\right]\hat{\Lambda}_{t,t+1} \quad \text{(B.48)}$$

or

$$\hat{x}_{\tau,t} = X_a^\tau\left(\hat{p}_t^w + \hat{a}_{\tau,t}\right) - X_w^\tau \hat{w}_{\tau,t} + X_\lambda^\tau \hat{\Lambda}_{t,t+1} + \beta\hat{x}_{\tau,t+1} \quad \text{(B.49)}$$

where

$$X^\tau \equiv \frac{1}{\kappa x_\tau}, \quad \text{(B.50)}$$

$$X_a^\tau \equiv X^\tau p^w a_\tau, \quad \text{(B.51)}$$

$$X_w^\tau \equiv X^\tau w_\tau, \quad \text{(B.52)}$$

$$X_\lambda^\tau \equiv \frac{\beta(1+\rho)}{2}. \quad \text{(B.53)}$$

Appendix C. Log-linearized system

According to equations (2.4), (2.6), (2.10), (2.13), (2.14), (2.15), (2.16), (2.17), (2.18), (2.19), (2.21), (2.22), (2.23), (2.24), (2.26), (2.31), (2.32), (2.33), (2.35), (2.37), (2.38), (2.45), (2.50), (2.51), (2.58), (2.59), (2.60), (2.67), (2.72), (2.73), (2.74), (2.75), (2.76), and (B.41), we obtain following log-linearized system. Note that $\tau = 1,2$, $\tau' = 1,2$, and $\tau \neq \tau'$.

$$\hat{u}_t = -\frac{n_1}{u}\hat{n}_{1,t-1} - \frac{n_2}{u}\hat{n}_{2,t-1} \quad \text{(C.1)}$$

$$\hat{k}_t = \hat{v}_t + \hat{k}_{t-1}^p \quad \text{(C.2)}$$

$$\hat{k}_t^p = \xi\hat{k}_{t-1}^p + (1-\xi)\hat{\imath}_t \quad \text{(C.3)}$$

$$\hat{\lambda}_t = -\hat{c}_t \quad \text{(C.4)}$$

$$0 = \hat{\Lambda}_{t+1} + \hat{r}_{t+1} - \hat{\pi}_{t+1} \quad \text{(C.5)}$$

$$\hat{v}_t = \eta_v \hat{r}_t^k \quad \text{(C.6)}$$

$$\hat{\imath}_t = \frac{1}{1+\beta}\hat{\imath}_{t-1} + \frac{1}{(1+\beta)\eta_k}\hat{q}_t^k + \frac{\beta}{1+\beta}\hat{\imath}_{t+1} \quad \text{(C.7)}$$

$$\hat{q}_t^k = \beta(1-\delta)E_t\hat{q}_{t+1}^k + [1-\beta(1-\delta)]E_t\hat{r}_{t+1}^k - (\hat{r}_{t+1} - \hat{\pi}_{t+1}) \quad \text{(C.8)}$$

$$\hat{s}_{\tau,t} = \hat{m}_{\tau,t} - \hat{u}_t \quad \text{(C.9)}$$

$$\hat{m}_{\tau,t} = \sigma\hat{u}_t + (1-\sigma)\hat{v}_{\tau,t} \quad \text{(C.10)}$$

$$\hat{q}_{\tau,t} = \hat{m}_{\tau,t} - \hat{v}_{\tau,t} \quad \text{(C.11)}$$

$$\hat{y}_t = \hat{z}_t + \alpha\hat{k}_t + (1-\alpha)\hat{n}_t \quad \text{(C.12)}$$

$$\hat{n}_t = \omega_1\hat{n}_{1,t} + \omega_2\hat{n}_{2,t} \tag{C.13}$$
$$\hat{z}_t = \rho^z\hat{z}_{t-1} + \hat{\zeta}_t^z \tag{C.14}$$
$$\widehat{w}_t = \omega_1\widehat{w}_{1,t} + \omega_2\widehat{w}_{2,t} \tag{C.15}$$
$$\hat{x}_{\tau,t} = \hat{q}_{\tau,t} + \hat{v}_{\tau,t} - \hat{n}_{\tau,t-1} \tag{C.16}$$
$$\hat{n}_{\tau,t} = (1-\rho)\hat{x}_{\tau,t} + \hat{n}_{\tau,t-1} \tag{C.17}$$
$$\hat{\Lambda}_{t,t+1} = \hat{\lambda}_{t+1} - \hat{\lambda}_t \tag{C.18}$$
$$\hat{r}_t^k = \hat{p}_t^w + \hat{y}_t - \hat{k}_t \tag{C.19}$$
$$\hat{a}_t = \hat{y}_t - \hat{n}_t \tag{C.20}$$
$$\hat{a}_{\tau,t} = \hat{a}_t + \hat{n}_t - \hat{n}_{\tau,t} \tag{C.21}$$
$$\hat{b}_t = \hat{k}_t^p \tag{C.22}$$
$$\hat{\pi}_t^w = \widehat{w}_t - \widehat{w}_{t-1} + \hat{\pi}_t \tag{C.23}$$
$$\hat{\pi}_{\tau,t}^w = \widehat{w}_{\tau,t} - \widehat{w}_{\tau,t-1} + \hat{\pi}_t \tag{C.24}$$
$$\hat{\chi}_{\tau,t} = -(1-\chi_\tau)[\hat{\mu}_{\tau,t} - \hat{\epsilon}_{\tau,t}] \tag{C.25}$$
$$\hat{\epsilon}_{\tau,t} = \beta\lambda_\tau\rho(E_t\hat{\Lambda}_{t,t+1} - E_t\hat{\pi}_{t+1} + E_t\hat{\epsilon}_{\tau,t+1}) \tag{C.26}$$
$$\hat{\mu}_{\tau,t} = \beta\lambda_\tau x_\tau E_t\hat{x}_{\tau,t+1} - \beta\lambda_\tau x_\tau N_\tau\mu_\tau^2\left(\widehat{w}_{\tau,t} - \hat{\pi}_{t+1} - \widehat{w}_{\tau,t+1}\right) + \beta\lambda_\tau\left(\hat{\mu}_{\tau,t+1} - E_t\hat{\pi}_{t+1} + E_t\hat{\Lambda}_{t,t+1}\right) \tag{C.27}$$
$$\hat{\pi}_t = \varphi\hat{p}_t^w + \beta E_t\hat{\pi}_{t+1} \tag{C.28}$$
$$\widehat{w}_{\tau,t}^o = \phi_a^\tau\left(\hat{p}_t^w + \hat{a}_{\tau,t}\right) + \left(\phi_x^\tau + \phi_{\tau,s}^\tau\right)E_t\hat{x}_{\tau,t+1} + \phi_{\tau',s}^\tau E_t\hat{x}_{\tau',t+1} + \phi_b^\tau\hat{b}_t + \phi_{\tau,s}^\tau\hat{s}_{\tau,t+1} +$$
$$\phi_{\tau',s}^\tau\hat{s}_{\tau',t+1} + \left[\phi_{\tau,s}^\tau + \phi_{\tau',s}^\tau + \frac{\phi_x^\tau}{2}\right]E_t\hat{\Lambda}_{t,t+1} + \phi_\chi^\tau\hat{\chi}_{\tau,t} + \left[\phi_{\tau,s}^\tau(1-\chi_\tau)^{-1} - \phi_\chi^\tau\rho_\tau\beta\right]\hat{\chi}_{\tau,t+1} +$$
$$\phi_{\tau',s}^\tau(1-\chi_{\tau'})^{-1}\hat{\chi}_{\tau',t+1}. \tag{C.29}$$
$$\hat{x}_{\tau,t} = X_a^\tau\left(\hat{p}_t^w + \hat{a}_{\tau,t}\right) - X_w^\tau\widehat{w}_{\tau,t} + X_\lambda^\tau\hat{\Lambda}_{t,t+1} + \beta\hat{x}_{\tau,t+1} \tag{C.30}$$
$$\hat{y}_t = y_c\hat{c}_t + y_i\hat{\imath}_t + y_g\hat{g}_t + y_v\hat{v}_t + y_{1,x}(2\hat{x}_{1,t} + \hat{n}_{1,t-1}) + y_{2,x}(2\hat{x}_{2,t} + \hat{n}_{2,t-1}) \tag{C.31}$$
$$\hat{g}_t = \hat{y}_t \tag{C.32}$$
$$\hat{r}_{t+1} = \rho^s\hat{r}_t + (1-\rho^s)[r_\pi\hat{\pi}_t + r_y\hat{y}_t] \tag{C.33}$$
$$E_t\widehat{w}_{\tau,t} = \gamma_\tau\widehat{w}_{\tau',t} + \gamma_b^\tau\left(\widehat{w}_{\tau,t-1} - \hat{\pi}_t\right) + \gamma_o^\tau\widehat{w}_{\tau,t}^o + \gamma_{\tau,f}^\tau(E_t\widehat{w}_{\tau,t+1} + E_t\hat{\pi}_{t+1}) + \gamma_{\tau',f}^\tau(E_t\widehat{w}_{\tau',t+1} + E_t\hat{\pi}_{t+1}) \tag{C.34}$$

The endogenous variables are $\hat{a}_t$, $\hat{a}_{\tau,t}$, $\hat{b}_t$, $\hat{c}_t$, $\hat{\chi}_{\tau,t}$, $\hat{\epsilon}_{\tau,t}$, $\hat{g}_t$, $\hat{\imath}_t$, $\hat{k}_t$, $\hat{k}_t^p$, $\hat{\lambda}_t$, $\widehat{\Lambda}_{t,t+1}$, $\widehat{m}_{\tau,t}$, $\hat{\mu}_{\tau,t}$, $\hat{n}_t$, $\hat{n}_{\tau,t}$, $\hat{v}_t$, $\hat{p}_t^w$, $\hat{\pi}_t$, $\hat{\pi}_t^w$, $\hat{\pi}_{\tau,t}^w$, $\hat{q}_{\tau,t}$, $\hat{q}_t^k$, $\hat{r}_{t+1}$, $\hat{r}_t^k$, $\hat{s}_{\tau,t}$, $\hat{u}_t$, , $\hat{v}_{\tau,t}$, , $\widehat{w}_t$, , $\widehat{w}_{\tau,t}$, $\widehat{w}_{\tau,t}^o$, $\hat{x}_{\tau,t}$, $\hat{y}_t$, and $\hat{z}_t$. The exogenous shock is ζ_t^z.

References

Andolfatto, D., 1996. Business cycles and labor–market search. American Economic Review 86, 112–132.

Blanchard, O., Galí, J., 2010. Labor markets and monetary policy: A new Keynesian model with unemployment. American Economic Journal: Macroeconomics 2, 1–30.

Calvo, G.A., 1983. Staggered prices in a utility–maximizing framework. Journal of Monetary Economics 12, 383–398.

Christofides, L.N., Stengos, T., 2007. A non–parametric test of the symmetry of PSID wage–change distributions. Economics Letters 71, 363–368.

Devicienti, F., Maida, A., Sestito, P., 2007. Downward wage rigidity in Italy: Micro–based measures and implications. Economic Journal 117, F530–F552.

Diamond, P.A., 1982. Aggregate demand management in search equilibrium. Journal of Political Economy 90, 881–894.

Erceg, C.J., Henderson, D.W., Levin, A.T., 2000. Optimal monetary policy with staggered wage and price contracts. Journal of Monetary Economics 46, 281–313.

Galí, J., 2011. Unemployment Fluctuations and Stabilization Policies: A New Keynesian Perspective. MIT Press, Cambridge, MA.

Galí, J., 2012. Notes for a new guide to Keynes (I): Wages, aggregate demand, and employment. NBER Working Paper No. 18651.

Gertler, M., Karadi, P., 2011. A model of unconventional monetary policy. Journal of Monetary Economics 58, 17–34.

Gertler, M., Sala, L., Trigari, A., 2008. An estimated monetary DSGE model with unemployment and staggered nominal wage bargaining. Journal of Money, Credit and Banking 40, 1713–1764.

Gertler, M., Trigari, A., 2009. Unemployment fluctuations with staggered Nash wage bargaining. Journal of Political Economy 117, 38–86.

Hall, R.E., 2005. Employment fluctuations with equilibrium wage stickiness. American Economic Review 95, 50–65.

Kahn, S., 1997. Evidence of nominal wage stickiness from microdata. American Economic Review 87, 993–1008.

Knoppik, C., Beissinger, T., 2013. How rigid are nominal wages? Evidence and implications for Germany. Scandinavian Journal of Economics 105, 619–641.

Kollmann, R., 2004. Welfare maximizing monetary and fiscal policy rules, in: Computing in Economics and Finance 2004. Society for Computational Economics. No. 102.

Mattesini, F., Rossi, L., 2009. Optimal monetary policy in economies with dual labor markets. Journal of Economic Dynamics and Control 33, 1469–1489.

Merz, M., 1995. Search in the labor market and the real business cycle. Journal of Monetary Economics 36, 269–300.

Mortensen, D.T., 1982. Property rights and efficiency in mating, racing, and related games. American Economic Review 72, 968–979.

Pissarides, C.A., 1985. Short–run equilibrium dynamics of unemployment vacancies, and real wages. American Economic Review 75, 676–690.

Pissarides, C.A., 2000. Equilibrium Unemployment Theory. MIT Press, Cambridge, MA.

Shimer, R., 2005. The cyclical behavior of equilibrium unemployment and vacancies. American Economic Review 95, 25–49.

Thomas, C., 2008. Search and matching frictions and optimal monetary policy. Journal of Monetary Economics 55, 936–956.

Zanetti, F., 2007. A non–Walrasian labor market in a monetary model of the business cycle. Journal of Economic Dynamics and Control 31, 2413–2437.

Table1: Parameters

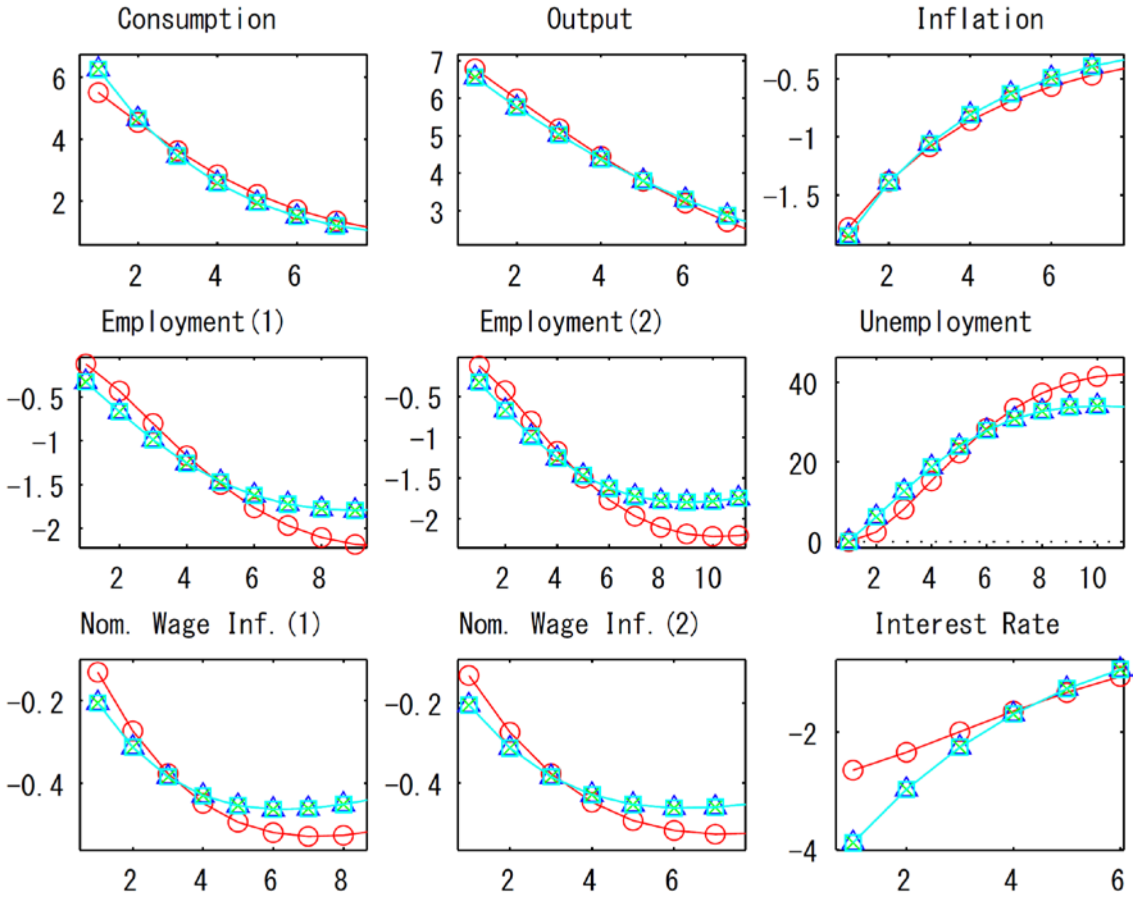
β	0.99	δ	0.025	σ	0.5	η	0.5	α	0.333	ρ^s	0.773
ϵ^g	0.2	λ_p	0.75	η_k	2.375	ψ_v	0.7	ρ	0.95	r_π	2.006
κ	2.81	ρ^z	0.9	ω_1	0.5	ϵ^p	6	r_y	0.332		

Table 2: Standard Deviations and Welfare Losses

	Homogeneous		Heterogeneous			
Policy Rule	(2.76)	(2.77)–(2.79)	(2.76)	(2.77)	(2.78)	(2.79)
$\hat{c}_t$	1.00	1.05	1.00	1.03	1.03	1.04
$\hat{n}_t$	1.00	0.82	1.00	0.80	0.79	0.80
$\hat{n}_{1,t}$	1.00	0.82	1.00	0.77	0.76	0.77
$\hat{n}_{2,t}$	1.00	0.82	1.00	0.82	0.81	0.82
$\hat{\pi}_t$	1.00	0.98	1.00	0.95	0.93	0.96
$\hat{y}_t$	1.00	1.01	1.00	1.02	1.03	1.01
$\hat{\pi}_t^w$	1.00	0.88	1.00	0.87	0.87	0.87
$\hat{\pi}_{1,t}^w$	1.00	0.88	1.00	0.85	0.85	0.85
$\hat{\pi}_{2,t}^w$	1.00	0.88	1.00	0.88	0.88	0.89
$\hat{r}_t$	1.00	1.21	1.00	1.12	1.12	1.17
$\Psi(\Omega = 0)$	1.00	1.10	1.00	1.04	1.04	1.08
$\Psi(\Omega = 1, \iota = 5)$	1.00	1.08	1.00	1.04	1.04	1.08
$\Psi(\Omega = 3.409, \iota = 0.276)$	1.00	0.96	1.00	0.98	0.98	1.02

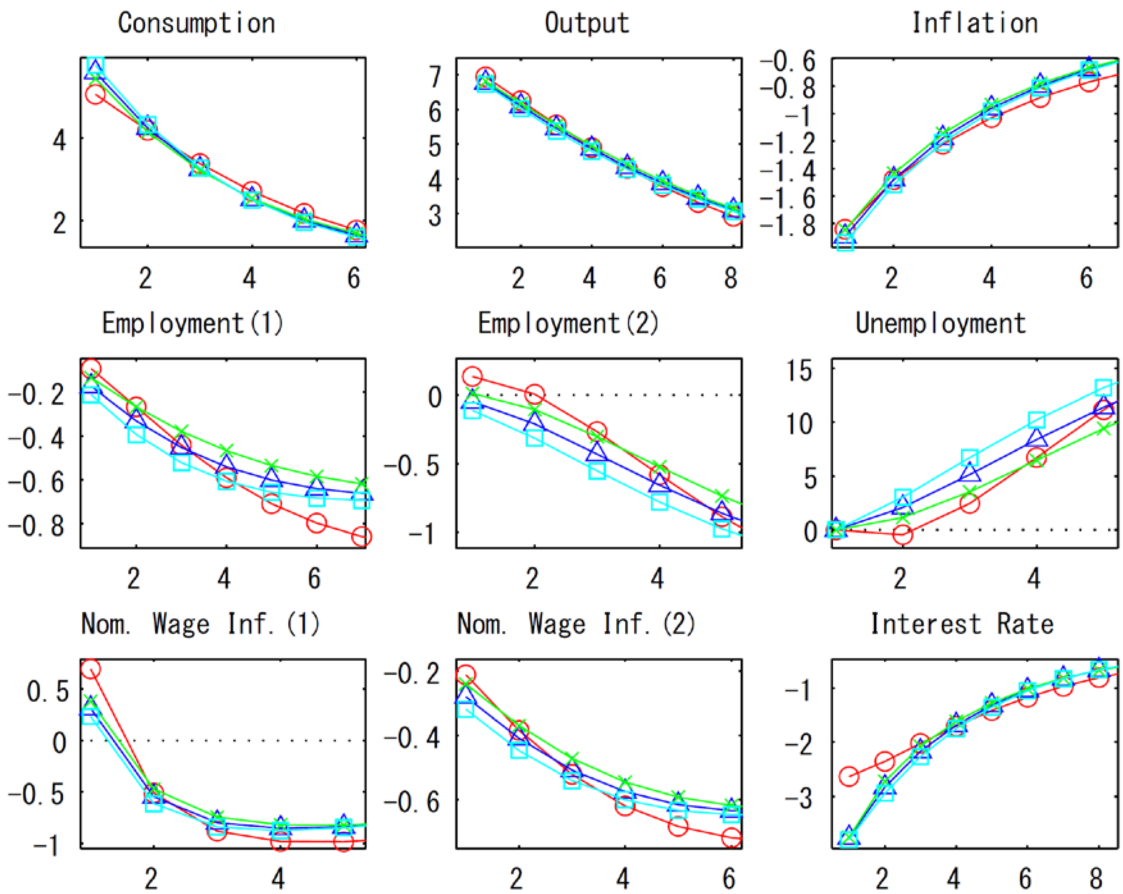
Notes: Standard deviations of respective variables and the welfare losses defined by (2.81) under alternative monetary rules (2.76), (2.77), (2.78), and (2.79). We normalize them by levels in the case with the rule (2.76).

Figure 1: Impulse Responses to the 1% Positive Shock on the Labor Productivity: Homogeneous Case ($\lambda_1 = \lambda_2$)



Notes: ○, ×, △, and □represent responses under policy rules (2.76), (2.77), (2.78), and (2.79), respectively. The vertical axis is in percent and the horizontal axis is the number of quarters.

Figure 2: Impulse Responses to the 1% Positive Shock on the Labor Productivity: Heterogeneous Case ($\lambda_1 < \lambda_2$)



Notes: ○, ×, △, and □represent responses under policy rules (2.76), (2.77), (2.78), and (2.79), respectively. The vertical axis is in percent and the horizontal axis is the number of quarters.